Part of Hints for Hw 3

Math 321

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2.3

1. $QQ^T = I$, so $det(QQ^T) = detI = 1$ and then $detQ * detQ^T = (detQ)^2 = 1$. $detQ = 1$ or -1

Second Part:Vector Calculus

1.1

1. $s(t) = \int_0^t \sqrt{4z^2 + 81z^4}dz = \int_0^t 2z\sqrt{1 + (81/4)z^2}dz = \frac{4}{81}\frac{2}{3}(1 + (81/4)z^2)^{3/2}|_0^t$. Solve for t and plug back to get the curve parametrized by s
3. a). $5 - 2 = 3$
b).$7 - 3 = 4$

1.2

1. cylindrical helix. Vector tangent to the curve is $\vec{v}(t) = -a\omega \sin \omega t\vec{e}_x + a\omega \cos \omega t\vec{e}_y + b\vec{e}_z$. The tangent line is $\vec{R}(u) = r(t) + u\vec{v}(t)$ where u is changing in this tangent line.
2. Ellipse. Plug in $\vec{r}_c = x_c\vec{e}_x + y_c\vec{e}_y$ $\vec{e}_1 = \cos\alpha\vec{e}_x + \sin\alpha\vec{e}_y$(You can also pick $-$ for sin). Then, we must have $\vec{e}_2 = -\sin\alpha\vec{e}_x + \cos\alpha\vec{e}_y$. Compare, and you can get x, y. Using $\cos^2\theta + \sin^2\theta = 1$ you can eliminate θ and get $f(x, y)$(α is OK since it's known.)
4. Just use the formula $L = \int_{\theta_1}^{\theta_2} |\frac{d\vec{r}}{d\theta}|d\theta$ and $S = \frac{1}{2}\int_{\theta_1}^{\theta_2} |\vec{r} \times \frac{d\vec{r}}{d\theta}|d\theta$
6. The first integral is 2π and the second one is 0
More:
b). $\frac{q_1q_2}{4\pi\varepsilon_0}(\frac{1}{2} - \frac{1}{3})$
c). (i). One possible answer $x = r\cos\theta, y = r\sin\theta$ and $\vec{r}(\theta) = r\cos\theta\hat{i} + r\sin\theta\hat{j}$
(ii). 0.
(iii). 2π. Actually, this is the same problem as the first integral in #6
(iv). $\frac{8}{3}r$
(v). $\vec{F} = 0$
(vi). $d\vec{r} = -r\sin\theta d\theta\vec{e}_x + r\cos\theta d\theta\vec{e}_y$. $d\vec{r} \times \vec{B} = -r\cos^2\theta\vec{e}_z d\theta$. The remaining work is easy.