Hw 7

Math 321

2.3

1. Suppose Q is an orthogonal matrix with size $n \times n$, prove that $det Q$ is either 1 or -1.

2. Suppose A is 3×3 matrix. $A_{ij} = 0$ if $i > j$. Show that $det A$ is product of the elements on the diagonal using both the definition $det(A) = \varepsilon_{ijk} A_{i1} A_{j2} A_{k3}$ and Laplace's expansion with repect to the first row.

Second Part:Vector Calculus

1.1

1. Let $\vec{r}(t) = [t^2, 3t^3]^T$. Find the arclength $s(t)$ and parametrize this curve using s.

2. For $\vec{r}(t)$, we have the arclength $s(t)$. $v = \frac{ds}{dt}$ is the speed and $\vec{v} = \frac{d\vec{r}}{dt}$ is the velocity. Use chain rule to differentiate $\vec{r}(t) = \vec{r}(s(t))$ and prove that $|\frac{d\vec{r}}{ds}| = |\frac{\vec{v}}{v}| = 1$.

3. $\vec{r}(t) = [x(t), y(t), z(t)]^T = [t^{\sqrt{73}}, \cos^{100}(2t) - 9\sin(\sqrt{7}t), \tan(5t^{1.2})]^T$. $s(t)$ is the arclength.

a). Find the integral $\int_2^5 |\frac{d\vec{r}}{ds}| ds$. What's the length of this curve between $s(t_1) = 2$ and $s(t_2) = 5$?

b). Find the integral $\int_3^7 \sqrt{(dx/ds)^2 + (dy/ds)^2 + (dz/ds)^2} ds$. What's the length of this curve between $s = 3$ and $s = 7$?

1.2

#1, #2 #4 #6

More:

a). Use equation (2) to convince yourself that $\int_C \vec{F} \cdot d\vec{r}$ is the work you learned in physics.

b). A charged particle with charge q_1 is located at the origin. Another particle B with charge q_2 is moving. The electric forces between the two particles acting on B is $\vec{F} = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2} \hat{r}$. Suppose particle B is moving from $2\hat{i}$ to $3\hat{k}$ along one special curve. Find the work the electric force did during this process.

c). (i). Consider the circle centered at the origin with radius r in $2D$ plane. Suppose a particle is moving counterclockwisely from $A(r, 0)$ along the circle. Parametrize the trajectory of the particle for one ratation ($\vec{r} = x\hat{i} + y\hat{j}$).

(ii). If there is a force field in (i) which is $\vec{F}(\vec{r}) = F_0\hat{i}$, calculate the work done on the particle.

(iii). If the force field in (i) is $\vec{F}(\vec{r}) = \frac{-y}{x^2+y^2}\hat{i} + \frac{x}{x^2+y^2}\hat{j}$, do (ii) again.

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(iv). Also consider the circle. If the line density of the circle is $\lambda(\vec{r}) = |\frac{x}{|\vec{r}|}|^3$. The total mass of the circle would be $m = \int_C \lambda(\vec{r})|d\vec{r}|$. Get this total mass.

(v). The circle is charged with charge line density $\lambda \vec{r} = \lambda_0$ which is a constant. There is electric field which is tangent to the circle at every point and the strength is $|\vec{E}| = |x|^2 + |y|^3$.The direction is counterclockwise. Calculate the total electric force acting on the circle.

(vi). Consider that the circle is a circuit with current I in flowing counterclockwisely. We have magnetic field $\vec{B} = \frac{x}{r}\vec{e}_x$. The total force would be $\vec{F} = \int_C (I d\vec{r}) \times \vec{B}$. Calculate this force.