

HW 6

Math 321

1.12

1. a). Convince yourself that Poisson vector has nothing to do with the motion of the center of the mass. It's only related to the motion about the center of mass.
- b). Consider a ball is rotating about x -axis and it's center is fixed at the origin. The radius of this ball is $3m$. At t_1 , the largest speed of the points on the surface of this ball is $6m/s$. Find the expression of the Poisson vector at time t_1 . Assume the basis is the standard basis $\{\hat{i}, \hat{j}, \hat{k}\}$.

2.1

Assume we have one right-handed orthonormal basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ for \mathbb{E}^3 . Also we have one new orthonormal basis $\{\vec{e}'_1, \vec{e}'_2, \vec{e}'_3\}$. The angles between \vec{e}'_1 and \vec{e}_1, \vec{e}_2 are $\pi/3, \pi/6$ respectively. The angle between \vec{e}'_2 and \vec{e}_3 is $\pi/4$. $\vec{e}'_2 \cdot \vec{e}_2 > 0$.

- a). Find the direction cosines for \vec{e}'_1 with respect to the old basis.
- b). Find the components of \vec{e}'_2 with respect to the old basis.
- c). If the new basis is right-handed, find the coordinate of \vec{e}'_3 with respect to the old basis.
- d). Give the expression of the Q matrix defined in the notes in this section. What are the 1st, 2nd and 3rd rows?
- e). Check that $QQ^T = I$ where I is the identity matrix. Such matrix is called orthogonal matrix.
- f)(*). Does the answer to e) have anything to do with the fact that both basis are orthonormal?
- g)(*). Let $[x_1, x_2, x_3]^T$ be the coordinate of \vec{x} with respect to the old basis. We have the equations: $\vec{e}'_j \cdot \vec{x} = j$. Write out these equations explicitly. Can you write them in matrix form? Can you solve these equations by just taking the inverse of a suitable matrix?

2.2

#2 #3 #4 #5 #7 #8 #10 #11(*)

More: $\vec{a} = [2, 3, 4]^T, \vec{b} = [1, 0, 0]^T, \vec{c} = [0, 2, 4]^T$. Use the Gram-Schmidt procedure to orthonormalize them.