# Hw 5

Math 321

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*This set of homework is more relevant to physics. Oops!*

## Exercises in 1.10

1. If $\vec{a}(t) = (e^{\tan t}\sec^2(t), \sqrt{t^2+1}, \frac{1}{(1+t^2)^2})$ and $\vec{v}(0) = 0$, get the expression of $\vec{v}$.

2. A particle with charge $q$ is moving in uniform magnetic field $\vec{B}$ with initial velocity $\vec{v}_0$, where $\vec{B} \perp \vec{v}_0$. Assume the Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$ is the only force on the particle. Show that the particle is doing uniform rotation.

3. The exercise in "Uniform rotation" starting with $\triangleright$

4. Do the same problems if $\vec{\omega}$ is not a constant. (The direction is fixed).

5. Let's look at one application of the two conservation laws in central force problem. *When I was in high school, I was very interested in such problems.*

   We know the gravitational force between two masses $m_1, m_2$ that acts on $m_1$ is

   $$\vec{F} = \frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$

   where $\hat{r}_{12}$ points to $m_2$. Now consider a planet which is orbiting around the sun. The mass of this planet is $m$ and the mass of the sun is $M$. The orbit of the planet is an ellipse. Since $M \gg m$, we can consider the sun to be fixed. The force here on the planet is thus a central force, and then we have the two conservation laws.

   a). If the potential energy $V(r)$ corresponding to the gravitational force at infinity is chosen to be 0, find the expressions of $F(r)$ and $V(r)$.
   b). Suppose the smallest distance between the sun and the planet is $r_0$ and the speed here is $v_0$. Derive the the largest distance $r_1$ between the sun and the planet and the speed there $v_1$.
   c)(*) Show that we must have $v_L = \sqrt{\frac{GM}{r_0}} < v_0 < \sqrt{\frac{2GM}{r_0}} = v_U$. ($v_L$ and $v_U$ are called the first and second cosmic speed respectively)
   d)(*) Use your force formula for rotation to argue that we can NOT have $\frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0} = m\omega_0 r_0^2$ at the point where the planet is closest to the sun.

## Exercises in 1.11

1. The exercise in this section.

2. A mass $M$ with charge $q$ was put at the origin in a uniform electic filed $\vec{E} = E\hat{x}$ when $t = 0$. Assume the initial velocity was 0. At some time $t_1$, the mass was splitted into two particles with different charges. At a later time $t_2$, one of them with mass $m$ was found to be at $2\hat{y}$. We assume $\{\hat{x}, \hat{y}, \hat{z}\}$ form an orthonormal right-handed basis. Find the position vector of the other particle at time $t_2$ if the electic forces were the only forces.

*I'll give problems for 1.12 next time.*