Hw 3

Math 321

Without declaration, we use Einstein's summation convention from now on.

Exercises in 1.6

1. #2

2. (a). Simplify $\delta_{ij}\delta_{jk}a_k$ and for each step write out the corresponding expression with $\sum$ notation.
   (b). We know $\delta_{11}^2 = \delta_{11}, \delta_{12}^2 = \delta_{12}$ etc, but why can't we have $\delta_{ij}\delta_{ij} = \delta_{ij}$?
   (c). By Einstein's convention, does $i\vec{e}_i$ mean $\vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3$? Why? If you answer 'No', then how can you simplify $\vec{a} \times \vec{b}$ using Einstein's convention, where $\vec{a} = 2\vec{e}_1 + 2\vec{e}_2 + 2\vec{e}_3$ and $\vec{b} = \vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3$?

3. Simplify the following expressions.
   (a). $\delta_{ij}\epsilon_{ikl}a_j$
   (b). $\epsilon_{ijk}\epsilon_{jlm}a_l b_m$ where $a_l = 2l, b_m = m^2$
   (c). $\epsilon_{kji}a_i b_j \epsilon_{klm}\delta_{mp}\epsilon_{plq}c_q$
   (d). $\vec{a} = a_i\vec{e}_i$ $\vec{b} = b_i\vec{e}_i$ $\vec{c} = c_i\vec{e}_i$ $\vec{d} = d_i\vec{e}_i$. Here $\{\vec{e}_i\}$ is an orthonormal basis. Simplify $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ and $((\vec{a} \times \vec{b}) \times \vec{c}) \cdot \vec{d}$

Exercises in 1.7

1. Given $\vec{a}, \vec{b}, \vec{c}$ below, determine if they are linearly independent. If they are $L.I.$, then in this order, are they right-handed or left-handed? (Hint: you only need to calculate $det(\vec{a}, \vec{b}, \vec{c})$ once to answer both questions.)
   (a). $\vec{a} = (2, 1, 3), \vec{b} = (0, 1, 0), \vec{c} = (4, 5, 2)$
   (b). $\vec{a} = (2, 1, 3), \vec{b} = (4, 5, 3), \vec{c} = (0, 2, 0)$
   (c). $\vec{a} = (2, 1, 3), \vec{b} = (4, 5, 3), \vec{c} = (3, 3, 3)$

2. #1 #3 #4 #6 #7

Other problems in this section are also interesting, so I'll pick some of them in the next set of homework.