Part of anwsers to Hw 2

Math 321

The problems with $*$ are for the ones who like math.

Section 1.4 #1: $\vec{e}_j \cdot \vec{w} = \sum_{i=1}^{3} w_i \delta_{ji} = w_j$

#4: $\sum_{i=1}^{3}\sum_{j=1}^{3} v_i w_j \vec{a}_i \cdot \vec{a}_j$

#5: (i) a_i (ii) $a_1b_1 + a_2b_2 + a_3b_3$ (iii) 3

Extra (a). Component form is $|\sum_{i=1}^{n} x_i y_i| \le \sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}$

Extra (c). The component corresponding to $\cos 3x$ is $\frac{1}{\pi}\int_0^{2\pi} f(x)\cos(3x)dx$.

Section 1.5. #2. Algebraically. $\vec{b} \times \vec{c} = \vec{b} \times (-\vec{a} - \vec{b}) = -\vec{b} \times \vec{a} = \vec{a} \times \vec{b}$. You can prove similarly that $\vec{c} \times \vec{a}$ is also equal to them.

Geometrically, we can arrange them into a loop–The end of the first one is the same as the head of the second one. Then, you can check that the cross products all have the same direction. The magnitude is twice the area of the triangle.

Using these identities, it's quite easy to get the law of sines.(Just divide by $|a||b||c|$).

#3. Algebra: $\vec{b} \times \vec{a} = (\vec{a} \times \vec{x}) \times \vec{a} = |\vec{a}|^2\vec{x} - (\vec{x} \cdot \vec{a})\vec{a}$. Simplify it and you get the answer.

Geometrically, $\vec{x}$ must be in the plane perpendicular to $\vec{b}$. $|x|\sin\theta$ is a constant. You can see easily that the component $\vec{a}_\perp$ is fixed and the component $\vec{a}_\parallel$ is arbitrary.