Some Hints for Hw 15

Math 321

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Easy problems

1. Ans: a). $e^{3\pi i/2}$ b). $2e^{-i\pi/3}$ c). $e^{i\pi}$ d). $\frac{1}{2}e^{i0}$

2. a.IMPORTANT CONCLUSION. Figure out!
   b. The answers to them all are 'NO'.

3. The hint is just to get $z^n = re^{i\theta}$. Assume $z = \rho e^{i\alpha}$ find ρ and all possible α.

4. Determine if the integral of the fucntion over $C : |z| = 1$ is zero or not.
   a). $f(z) = z^{1000} + \sin z$—(YES)
   b). $f(z) = \frac{\cos z}{z+3}$—(YES)
   c). $f(z) = \frac{1}{(4z^2+1)(z-7)}$—(NO)
   d). $f(z) = \frac{8}{z^2-z+1/4}$—(YES)

5. ('Extra' omitted)

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1. Solve $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$:
   For trig: $x = \tan\theta$ and then you'll get $\int_{-\pi/2}^{\pi/2} \frac{1+\cos(2\theta)}{2} d\theta = \frac{\pi}{2}$
   For complex, just refer to the notes. (We can see that real techniques are tricky while complex techniques recover the intrinsic properties of that function.)

2. #3
   Method is similar. $z^4 + z^2 + 1 = 0$. $z^2 = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i = e^{i2\pi/3}, e^{i4\pi/3}$ and then you can see that your singularities would be $e^{i\pi/3}, e^{i4\pi/3}, e^{i2\pi/3}, e^{i5\pi/3}$. That means the denominator can be factored as $(z - e^{i\pi/3})(z - e^{i2\pi/3})(z - e^{i4\pi/3})(z - e^{i5\pi/3})$

3. #5: Nice challenging problem. Follow the hint on the notes. You should know how to get $\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. The answer is $\int_{-\infty}^{+\infty} \cos x^2 dx = \int_{-\infty}^{+\infty} \sin x^2 dx = \sqrt{\frac{\pi}{2}}$