Hw 15(Two pages)

Math 321

Easy problems

1. Write those complex numbers in polar form.
   a). $-i$ b). $1-\sqrt{3}i$ c). -1 d). $1/2$

2. a. If $f(z)=f(x+iy)=u(x,y)+iv(x,y)$ is analytical, show that both u and v are harmonic functions.
   b. For the following u, is it possible to find the conjugate function v such that $u+iv$ is analytical?
   $u(x)=4\tan^3(x)$, $u(x)=3x^2y+9xy^{100}$, $u(x,y)=g(x+y)$ where $g''(u)$ is nice and nonzero.

3. Solve the following equations:
   a). $z^3+8=0$
   b). $z^4+6z^2+8=0$
   c). $3z^2=i$

4. Determine if the integral of the fucntion over $C:|z|=1$ is zero or not.
   a). $f(z)=z^{1000}+\sin z$
   b). $f(z)=\frac{\cos z}{z+3}$
   c). $f(z)=\frac{1}{(4z^2+1)(z-7)}$
   d). $f(z)=\frac{8}{z^2-z+1/4}$

5. Parametrize these:
   a). The graph of $f(x)=\frac{1}{x^2+1}$. Find the formula to calculate length.
   b). The surface of the Earth inside the Tropic of Cancer. Find the area element.(You can use R to mean the radius of the earth).
   c). The area inside the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the area element and write out the integral for the area.
   d). The cylinder under $z=\sqrt{x^2+y^2}$ and above $z=-(x^2+y^2)$ centered at $x=y=0$ with radius 1. Find the volume.
   e). Parametriz the surface of $z=h(x,y)=x^2-y^2$. Are your parameters orthogonal parameters?

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1. Solve $\int_{-\infty}^{\infty}\frac{1}{(x^2+1)^2}dx$ using trig-substitution and complex integral both. Compare these two methods.

2. #3
3. #5