Easy problems

1. Write those complex numbers in polar form.
   a). $-i$  
   b). $1 - \sqrt{3}i$  
   c). $-1$  
   d). $1/2$

2. a. If $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ is analytical, show that both $u$ and $v$ are harmonic functions.
   b. For the following $u$, is it possible to find the conjugate function $v$ such that $u + iv$ is analytical?
      $u(x) = 4 \tan^3(x), \quad u(x) = 3x^2y + 9xy^{100}, \quad u(x, y) = g(x + y)$ where $g''(u)$ is nice and nonzero.

3. Solve the following equations:
   a). $z^3 + 8 = 0$
   b). $z^4 + 6z^2 + 8 = 0$
   c). $3z^2 = i$

4. Determine if the integral of the function over $C : |z| = 1$ is zero or not.
   a). $f(z) = z^{1000} + \sin z$
   b). $f(z) = \frac{\cos z}{z+3}$
   c). $f(z) = \frac{1}{(4z^2+1)(z-7)}$
   d). $f(z) = \frac{8}{z^2 - z + 1/3}$

5. Parametrize these:
   a). The graph of $f(x) = \frac{1}{x^2+1}$. Find the formula to calculate length.
   b). The surface of the Earth inside the Tropic of Cancer. Find the area element. (You can use $R$ to mean the radius of the earth). 
   c). The area inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Find the area element and write out the integral for the area.
   d). The cylinder under $z = \sqrt{x^2 + y^2}$ and above $z = -(x^2 + y^2)$ centered at $x = y = 0$ with radius 1. Find the volume.
   e). Parametrize the surface of $z = h(x, y) = x^2 - y^2$. Are your parameters orthogonal parameters?

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1. Solve $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$ using trig-substitution and complex integral both. Compare these two methods.
2. #3
3. #5