

Hw 14

Math 321

3.2

1. #4, #7
2. Calculate $\int_{|z|=4} \frac{1}{(z+1)(z-1)(z+2i)} dz$ without calculating the partial fraction expressions.
3. Calculate $\int_{|z|=2} \frac{1}{z(z+3)^2} dz$ and $\int_{|z|=2} \frac{1}{z^2(z+3)} dz$
4. Calculate $\int_{|z|=3} \frac{e^z}{(z-2)^3} dz$
5. Calculate $\int_{|z|=2} \frac{\sin z}{z^2+1} dz$
6. (*) (Challenging problems)
(This one could be really hard for you) Calculate $\int_{|z|=1/2} \frac{e^z}{\sin(2z)} dz$

4

1. Calculate $\int_0^{2\pi} \frac{1}{2-\sin\theta} d\theta$ (Hint: On unit circle, $\sin\theta = \frac{z-z^{-1}}{2i}$)
2. #1 (Just consider the case $a > b > 0$)
3. Redo the integral $\int_{-\infty}^{+\infty} \frac{1}{x^4+1} dx$ to make sure you understand.
4. Calculate $\int_0^{+\infty} \frac{\cos x}{x^2+1} dx$. I think I need to give you a little hint:
First of all, notice that this integral equals $\frac{1}{2} \int_{-\infty}^{\infty} \dots dx$

$$\frac{\cos x}{x^2+1} = \operatorname{Re}\left(\frac{e^{iz}}{z^2+1}\right)$$

on $x - axis$. Thus, you can calculate the integral:

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{x^2+1} dx$$

first and then take the real part. Then, you calculate:

$$\int_C \frac{e^{iz}}{z^2+1} dz$$

where C is the interval $[-R, R]$ with the upper semi-circle. You need to prove the integral on the upper semi-circle goes to zero.