3.2

1. #4, #7
2. Calculate \( \int_{|z|=4} \frac{1}{(z+1)(z-1)(z+2i)} \, dz \) without calculating the partial fraction expressions.
3. Calculate \( \int_{|z|=2} \frac{1}{z(z+3)^2} \, dz \) and \( \int_{|z|=2} \frac{1}{ze^z} \, dz \)
4. Calculate \( \int_{|z|=3} \frac{e^z}{(z-2)^2} \, dz \)
5. Calculate \( \int_{|z|=2} \frac{\sin z}{z^2+1} \, dz \)
6. (*) (Challenging problems)
   (This one could be really hard for you) Calculate \( \int_{|z|=1/2} \frac{e^z}{\sin(2z)} \, dz \)

4

1. Calculate \( \int_{0}^{2\pi} \frac{1}{2-\sin \theta} \, d\theta \) (Hint: On unit circle, \( \sin \theta = \frac{z-z^{-1}}{2i} \))
2. #1 (Just consider the case \( a > b > 0 \))
3. Redo the integral \( \int_{-\infty}^{\infty} \frac{1}{x^2+1} \, dx \) to make sure you understand.
4. Calculate \( \int_{0}^{\infty} \frac{\cos x}{x^2+1} \, dx \). I think I need to give you a little hint:
   First of all, notice that this integral equals \( \frac{1}{2} \int_{-\infty}^{\infty} \ldots \, dx \)
   \[
   \frac{\cos x}{x^2+1} = Re\left(\frac{e^{iz}}{z^2+1}\right)
   \]
on \( x - axis \). Thus, you can calculate the integral:
   \[
   \int_{-\infty}^{\infty} \frac{e^{ix}}{x^2+1} \, dx
   \]
   first and then take the real part. Then, you calculate:
   \[
   \int_{C} \frac{e^{iz}}{z^2+1} \, dz
   \]
   where \( C \) is the interval \([-R, R]\) with the upper semi-circle. You need to prove the integral on the upper semi-circle goes to zero.