Part of Hints for Hw 13

Math 321

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1.5

1. Use the ratio test. Center is 1/2. Radius is 1. Then the domain of convergence is a disk with radius one centered at 1/2
2. #4: The series is $\sum_{n=0}^{\infty}(-z^2)^n = \sum_{n=0}^{\infty}(-1)^n z^{2n}$. Radius of convergence is 1.
#5: $\frac{1}{z} = \frac{1}{2+(z-2)} = \frac{1}{2}\frac{1}{1+(z-2)/2} = \frac{1}{2}\sum_{n=0}^{\infty}(-\frac{z-2}{2})^n = \ldots$

1.6

#2: (I'm not doing using series.) $\exp(i) = \cos 1 + i\sin 1$.
$\cos(i) = \frac{1}{2}(e^{i*i} + e^{-i*i}) = \frac{1}{2}(e^{-1} + e)$ and you can get similar expression for $\sin(i)$
#4: $\exp(-i\bar{z})$
#5: We have $(35) + i(36) = \sum_{n=0}^{N} e^{inx} = \frac{1-e^{iNx}}{1-e^{ix}} = \frac{(1-e^{iNx})(1-e^{-ix})}{2-2\cos x}$. The real part of this expression is (35) and the imaginary part is (36)

1.7+1.8

1. $1+\sqrt{3}i = 2e^{i\pi/3}$ and $i = e^{i\pi/2}$. Square roots for the first: $\sqrt{2}e^{i\pi/6}$ and $\sqrt{2}e^{i7\pi/6}$. For the second: $\sqrt{i} = \{e^{i\pi/4} = \frac{\sqrt{2}}{2}(1+i), e^{i5\pi/4} = \frac{\sqrt{2}}{2}(1-i)\}$
2. Calculate $\ln(1+\sqrt{3}i) = \ln 2 + i\pi/3$ and $\ln(i) = i\pi/2$

(**The following problems are more important**)

2.1

Given $u(x,y)$ find the conjugate function $v(x,y)$ such that $u(x,y)+iv(x,y)$ is analytical (namely $u(x,y)+iv(x,y)$ can be written as a function $f(z)$ and $f'(z)$ exists in the domain we are interested in). Find $f(z)$.
a). $u = x + y$
b). $u = 2x^2 - 2y^2 + 2x + 3$

c). $u = e^x \cos(y)$
Ans: All we need is Cauchy-Riemann formula. I'll do b) as an example.
a). $v = y - x + C$ and thus $u + iv = z - iz + iC = (1 - i)z + C'$
b). $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 4x + 2$. Then $v(x, y) = 4xy + 2y + g(x)$. $\partial v/\partial x = 4y + g'(x) = -\partial u/\partial y = 4y$. $g'(x) = 0$ and thus $g(x) = C$. $u + iv = 2x^2 - 2y^2 + i4xy + 2x + 2iy + 3 + C'$. The highest order is $2x^2$ and we thus guess we have $2z^2$. Subtract this and you'll have only $2(x + iy) + 3 + C'$. Ans is $2z^2 + 2z + 3 + C'$ where C' is pure imaginary.
c). $e^z + iC$

3.1

1. #1: $4\pi i$
#5: $2i\pi/3$
#8: No.
$\Big((*)\#10\Big)$: Think about where the power series fails.
2. Calculate $\int_{|z|=2} \bar{z} dz = 8\pi i$
3.(*)(For smart guys)$\int_{|z|=1} \frac{\sin z}{z} dz = 0$.
$\int_{|z|=2} \frac{\sin i}{z^2+1} dz$. Hint: $\frac{1}{z^2+1} = \frac{1}{2i}(\frac{1}{z-i} - \frac{1}{z+i})$