Hw 12

Math 321

Some easy problems

1. Consider $f(x, y, z)$, we know $df = d\vec{r} \cdot \nabla f$.
a). Get the expression of df explicitly using Catesian coordinates.
b). If $x = x(t), y = y(t), z = z(t)$, get $\frac{df}{dt}$
2. Consider $\vec{v}(x, y, z) = x^2\vec{e}_x$
a). Calculate $\nabla \cdot \vec{v}$
b). Write $\vec{v}$ as $u(r, \theta, \varphi)\hat{r} + v(r, \theta, \varphi)\hat{\theta} + w(r, \theta, \varphi)\hat{\varphi}$ where we use spherical coordinates now.
c). Use the fact that $\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r \sin\theta}\frac{\partial}{\partial \varphi}$ to get $\nabla \cdot \vec{v}$ in spherical coordinate form. Compare your answer to a).

Challenging problems

3. Mimic what we did for spherical coordinate to get ∇ in cylindrical coordinate form.
4. Get $\Delta = \nabla \cdot \nabla$ in spherical coordiante form. (You know in Cartesian form $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$)

3.6

(*) For smart guys: Calculate $\int_S x^2 dS$ where S is the sphere $x^2 + y^2 + z^2 = 4$.
Hint: use the fact that $\int_S x^2 dS = \int_S y^2 dS = \int_S z^2 dS$ if the region is symmetric. Add them together and use Gauss's theorem!

3.7

#1 #8 (Do them in two ways to see how the theorems work.)
#1 #2 in 'Gradient' notes.
Problems for complex numbers in next page.

Part 3 Complex Calculus

1.1

All problems.

1.3

1. When does this series converge $\sum_{n=1}^{\infty} \frac{z^n \cos(n\pi)}{e^n}$? When it converges, what is the sum?

2. Calculate $\sum_{n=1}^{N} \cos(n\theta)$ and $\sum_{n=1}^{N} \sin(n\theta)$ by noticing that if $z = e^{i\theta}$, then $\cos(n\theta) + i\sin(n\theta) = e^{in\theta} = z^n$