Solutions to Quiz 2

Below $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = r\hat{r}$ is a function of a parameter t.
1). True or false? Explain. a. $\dot{\vec{r}} = \dot{r}\hat{r}$. b. $|d\vec{r}/dt| = d|\vec{r}|/dt = \dot{r}$ (6')
a. False. Right hand side is parallel to $\vec{r}$ while the derivative of $\vec{r}$ (or velocity) doesn't have to be. More rigorously:

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\frac{d}{dt}\hat{r}$$

The second term is not necessarily zero. Example, the uniform rotation, the second is nonzero. Actually, one can even check $\frac{d}{dt}\hat{r}\perp\hat{r}$. This means the first term is the component parallel to $\vec{r}$ while the second term is perpendicular to $\vec{r}$. The problem throws away the perp part.
b. False. Left hand side is speed while the right hand side is just the changing rate of r. Actually, $\dot{r}$ is just the radius part of $d\vec{r}/dt$. Namely, we have:

$$\dot{r} = \hat{r}\cdot\frac{d}{dt}\vec{r}$$

This can be verified by the formula we derived in (a). One can even check:

$$|\frac{d}{dt}\vec{r}| = \sqrt{(\dot{r})^2 + r^2|d\hat{r}/dt|^2}$$

2). Writing $\dot{\vec{r}} = \vec{v} = v_r\hat{r} + \vec{v}_\perp$, $v_r = \vec{v}\cdot\hat{r}$. Then show $\dot{r} = v_r$ using $\vec{r}\cdot\vec{r} = r^2$. (4')
Soln: First way is to use what we get in $1.a$.

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\frac{d}{dt}\hat{r}$$

We have verified that $\hat{r}\cdot\frac{d}{dt}\hat{r} = 0$. We then just dot this expression with $\hat{r}$ and get:

$$\hat{r}\cdot\frac{d}{dt}\vec{r} = \dot{r}$$

The left hand side is v_r exactly.
Second way is to make use of $\vec{r}\cdot\vec{r} = r^2$. We take derivatives on both sides:

$$2\vec{r}\cdot\frac{d}{dt}\vec{r} = 2r\dot{r}$$

Divide by $2r$ and you get what you need.
(Bonus) If $\dot{\vec{r}} = \vec{\omega}\times\vec{r}$, show that $|\vec{r}|$ won't change. (Notice $\vec{\omega}$ also depends on t. In homework, you are even required to solve $\vec{r}(t)$ for the case where $\vec{\omega}$ is a constant.). (2')
Ans: To check it doesn't change, we just take derivative on it.

$$\frac{d}{dt}|\vec{r}|^2 = 2\vec{r}\cdot\frac{d}{dt}\vec{r} = 2\vec{r}\cdot(\vec{\omega}\times\vec{r}) = 0$$