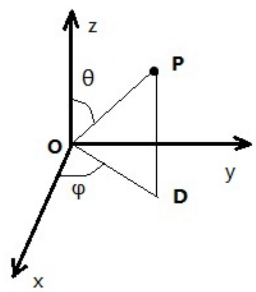
Solutions to Quiz 1

As in the figure below, $\overline{OD} = 1, \varphi = \pi/4, \theta = \pi/6$.
1). Write out the position vector of P in spherical representation (Don't use $\hat{x}, \hat{y}$ etc). (3')
2). Obtain the expression of $\hat{r}$ in terms of $\hat{x}, \hat{y}, \hat{z}$ (7')
3). (Bonus) Find the components of $\overrightarrow{OD}$ with respect to $\{\overrightarrow{OP}, \hat{z}\}$ (3')



Ans: 1). We can see that

$$\overline{OD} = \overline{OP}\sin\theta \Rightarrow \overline{OP} = 1/\sin(\pi/6) = 2$$

Thus, we have

$$\vec{r} = 2\hat{r}$$

(Note: $\hat{r}$ is a directional vector in spherical representation.)
2).

$$\hat{r} = \sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z} = \frac{\sqrt{2}}{4}\hat{x} + \frac{\sqrt{2}}{4}\hat{y} + \frac{\sqrt{3}}{2}\hat{z}$$

3). Actually, it's easy to see

$$\overrightarrow{OD} = \overrightarrow{OP} - \overrightarrow{DP} = \overrightarrow{OP} - 2 * \frac{\sqrt{3}}{2}\hat{z}$$

The components are 1 and $-\sqrt{3}$. (You can also draw a parallelogram to see similar relationship.)