Solutions to Quiz 1

As in the figure below, $\overrightarrow{OD} = 1, \varphi = \pi/4, \theta = \pi/6$.

1). Write out the position vector of $P$ in spherical representation (Don’t use $\hat{x}, \hat{y}$ etc). (3’)

2). Obtain the expression of $\hat{r}$ in terms of $\hat{x}, \hat{y}, \hat{z}$ (7’)

3). (Bonus) Find the components of $\overrightarrow{OD}$ with respect to $\{\overrightarrow{OP}, \hat{z}\}$ (3’)

Ans: 1). We can see that

$$\overrightarrow{OD} = \overrightarrow{OP} \sin \theta \Rightarrow \overrightarrow{OP} = \frac{1}{\sin(\pi/6)} = 2$$

Thus, we have

$$\hat{r} = 2\hat{r}$$

(Note: $\hat{r}$ is a directional vector in spherical representation.)

2).

$$\hat{r} = \sin \theta \cos \varphi \hat{x} + \sin \theta \sin \varphi \hat{y} + \cos \theta \hat{z} = \frac{\sqrt{2}}{4} \hat{x} + \frac{\sqrt{2}}{4} \hat{y} + \frac{\sqrt{3}}{2} \hat{z}$$

3). Actually, it’s easy to see

$$\overrightarrow{OD} = \overrightarrow{OP} - \overrightarrow{DP} = \overrightarrow{OP} - 2 \cdot \frac{\sqrt{3}}{2} \hat{z}$$

The components are 1 and $-\sqrt{3}$. (You can also draw a parallelogram to see similar relationship.)