Some Hints for HW2

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Points:
1. Linear combinations. Linearly independent. Basis, orthonormal basis. Components of vector with respect to a basis.
2. System of coordinate (vector basis+reference point). Position/radius vector ($\vec{r}$). Equations of lines and planes.

Hw2

3. A vector basis for a space is a set of linearly independent vectors. Any other vector in that space could be written as a linear combination of them. (One corollary is that the number of vectors in basis is the largest possible number of independent vectors in this space.)
5. Suppose the parallelogram is $ABCD$. The intersection of AC and BD is P. Let $\overrightarrow{AB} = \vec{a}_1$ and $\overrightarrow{AD} = \vec{a}_2$. We should represent $\overrightarrow{AP}$ in two independent ways.
The first way: $\overrightarrow{AP}$ is parallel to $\overrightarrow{AC}$. Then $\overrightarrow{AP} = t\overrightarrow{AC} = t(\vec{a}_1 + \vec{a}_2)$
The second way: It's $\overrightarrow{AB} + \overrightarrow{BP} = \vec{a}_1 + s\overrightarrow{BD} = \vec{a}_1 + s(-\vec{a}_1 + \vec{a}_2)$.
Setting them equal, we get:

$$t = 1 - s$$
$$t = s$$

This means $s = t = 1/2$. $\overrightarrow{AP} = \frac{1}{2}(\vec{a}_1 + \vec{a}_2)$ which means P is the midpoint of AC.
Similarly, you can show that P is the midpoint of BD

6. Label the three points as $A(1,2,3), B(2,3,1), C(3,2,1)$. Denote $\vec{a}_1 = \overrightarrow{AB}, \vec{a}_2 = \overrightarrow{AC}$.
Any point P in the plane can be determined by $\overrightarrow{AP} = s\vec{a}_1 + t\vec{a}_2$. However, if we don't put constraint on s, t, we just get the whole plane. It's obvious $s \geq 0, t \geq 0$.
We can see that the points on BC are special. They are the farthest we could go. Let Q be a point on the line segment BC. $\overrightarrow{AQ} = s'\vec{a}_1 + t'\vec{a}_2$. Because it's on BC, we then $\overrightarrow{AQ} = \vec{a}_1 + \lambda(-\vec{a}_1 + \vec{a}_2) = (1-\lambda)\vec{a}_1 + \lambda\vec{a}_2$. This means $s' + t' = (1-\lambda) + \lambda = 1$. For an arbitrary point inside the triangle, $s + t$ can only be smaller. Then, we should have $s + t \leq 1$.
To sum up:

$$s \geq 0$$
$$t \geq 0$$
$$s + t \leq 1$$

Then, we have the relationship:

$$\begin{aligned}
&< x-1, y-2, z-3 >= s < 1,1,-2 > +t < 2,0,-2 > \\
&x = 1+s+2t \\
&y = 2+s \\
&z = 3-2s-2t
\end{aligned}$$

with the constraint above

7. The line AB could be described by:

$$\vec{r} = \vec{r}_A + t(\vec{r}_B - \vec{r}_A)$$

To show that point is on this line, we just need to show that the vector satisfies this equation. Compare the coefficients, and we want to try $t = \beta/(\alpha+\beta)$. Plug this in and we get:

$$\vec{r} = \vec{r}_A + \frac{\beta}{\alpha+\beta}(\vec{r}_B - \vec{r}_A) = \frac{\alpha\vec{r}_A + \beta\vec{r}_B}{\alpha+\beta}$$

Sec 1.2

1. (i) That should be the parallelogram(the points inside are included) determined by these two vectors.

(ii) You could get the boundary lines first. Consider $\alpha = \beta$, $\alpha = -\beta$, $\alpha = 1$, $\alpha = -1$. Then, you get four lines. The regions are the ones bounded by these four lines.

7. Writing $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$, $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$. Plug these two into the equation, and you would get:

$$3\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AC} = 0$$

This implies $\overrightarrow{AO} = \frac{1}{3}(\overrightarrow{AB} + \overrightarrow{AC})$

8. Suppose we extend AO and it intersects BCD at P. We would like to show that P is the center.

Write $\overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{PA}$. Do similar things for $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$. You would then get:

$$(4\overrightarrow{OP} + \overrightarrow{PA}) + (\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}) = 0$$

We can see that the first term is parallel to PA while the second term is in plane BCD. The only possibility is that they are both 0. You can then derive other conclusions.