This morning in Section 343, some people were still confused with the completeness of the list of solutions. Let me try to reform that statement here again.

Claim 1 *For a n-th order linear equation, we need n independent basis solutions to the homogeneous part. The superposition of them forms a complete list of solutions to the homogeneous part.*

Let's illustrate this with the following second order equation example:

Claim 2 *Consider $y'' + Ay' + By = 0$ where A, B are constants. If the characteristic equation $\lambda^2 + A\lambda + B = 0$ has two complex roots $\alpha \pm \beta i$ where $\beta \neq 0$, then we have two basis solutions $e^{\alpha x} \cos \beta x$, $e^{\alpha x} \sin \beta x$ and the list*

$$\{y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x) | C_1, C_2 \in \mathbb{R}\}$$

is the complete list of solutions.

- First of all, the list is a list of solutions. We obtain $e^{\alpha x} \cos \beta x$ and $e^{\alpha x} \sin \beta x$ from the characteristic polynomial and therefore they must be solutions. By superposition since the equation is linear and homogeneous, the list is a list of solutions.

- Secondly, we must argue that for any solution y^* to the equation, y^* belongs to the list. This argument relies on the existence and uniqueness theorem. We take this theorem for granted. The version of the theorem for a second order equation is: If the coefficients of the second order linear equation are continuous on the interval with left endpoint a, then it has a unique solution satisfying $y(a) = D$ and $y'(a) = E$.

  Now, let's take $a = 0$ for simplicity. Look at y^*. It has a value $y^*(0)$(this is the D in the theorem), and derivative, $(y^*)'(0)$(this is E in the theorem). Now, we claim that we can find $\bar{y}$ from the list so that $\bar{y}$ also has a value $y^*(0)$ at 0 and derivative $(y^*)'(0)$ at 0. To show this, we just list the equations

  $$\bar{y}(0) = C_1 + C_2 * 0 = y^*(0)$$
  $$\bar{y}'(0) = \alpha C_1 + C_2 \beta = (y^*)'(0)$$

  We can find C_1 and C_2. That means we have a function from the list that is a solution to the equation and has the same value and derivative as y^*. By the existence and uniqueness theorem, they must be equal, namely $y^* = \bar{y}$. Thus, y^* must belong to the list. The list is therefore complete.