I'll just solve without sketching the graph.
6.4#5
$y'' + 3y' + 2y = f(t)$, $y(0) = 0, y'(0) = 0$. $f = u_0 - u_{10}$.
Taking the Laplace Transform on both sides:

$$\mathcal{L}(y'')+3\mathcal{L}(y')+2\mathcal{L}(y) = (s^2Y - s*0-0)+3(sY-0)+2Y = (s^2+3s+2)Y$$

Right hand side is

$$\mathcal{L}(u_0) - \mathcal{L}(u_{10}) = \frac{1}{s} - \frac{e^{-10s}}{s}$$

We hence have

$$Y(s) = \frac{1-e^{-10s}}{s(s^2+3s+2)} = \frac{1}{s(s+1)(s+2)} - \frac{e^{-10s}}{s(s+1)(s+2)}$$

For the partial fraction

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{s+2}$$

Multiplying the denominator, and setting $s \to 0, s \to -1, s \to -2$, we have

$$A = \frac{1}{2}, B = -1, C = \frac{1}{2}$$

Noticing that e^{-10s} is the shifting in time with 10, we have

$$y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} - u_{10}[\frac{1}{2} - e^{-(t-10)} + \frac{1}{2}e^{-2(t-10)}]$$

6.4#10
$y'' + y' + \frac{5}{4}y = g(t), y(0) = 0, y'(0) = 0$. $g(t) = \sin(t)[u_0 - u_\pi]$
Similarly, as above, the Laplace Transform of the left hand side is

$$(s^2 + s + \frac{5}{4})Y(s)$$

Be sure what you should include if the initial values are nonzero.
For the right hand side, we have

$$\mathcal{L}(\sin(t)) - \mathcal{L}(u_\pi \sin(t)) = \frac{1}{s^2+1} - \mathcal{L}(-u_\pi \sin(t-\pi)) = \frac{1}{s^2+1} + e^{-\pi s}\frac{1}{s^2+1}$$

Hence, we have

$$Y(s) = \frac{1}{(s^2+1)((s+1/2)^2+1)}[1+e^{-\pi s}]$$

From here, we see that

$$y(t) = h(t) + u_\pi h(t-\pi)$$

where $h(t)$ is the inverse Laplace Transform of $\frac{1}{(s^2+1)((s+1/2)^2+1)}$

We do partial fraction again. The denominator are irreducible quadratic factors. You should have

$$\frac{1}{(s^2+1)((s+1/2)^2+1)} = \frac{A_1 s + B_1}{s^2+1} + \frac{A_2 s + B_2}{(s+1/2)^2+1}$$

. Comparing the powers, we have

$$A_2 + A_1 = 0$$
$$B_1 + A_1 + B_2 = 0$$
$$\frac{5}{4}A_1 + B_1 + A_2 = 0$$
$$\frac{5}{4}B_1 + B_2 = 1$$

We solve that (you can do Gauss-elimination)

$$A_1 = -16/17, B_1 = 4/17, A_2 = 16/17, B_2 = 12/17$$

Hence, we have

$$\mathcal{L}(h) = -\frac{16}{17}\frac{s}{s^2+1} + \frac{4}{17}\frac{1}{s^2+1} + \frac{16}{17}\frac{s+1/2}{(s+1/2)^2+1} + \frac{4}{17}\frac{1}{(s+1/2)^2+1}$$

We find

$$h(t) = -\frac{16}{17}\cos(t) + \frac{4}{17}\sin(t) + \frac{16}{17}e^{-t/2}\cos(t) + \frac{4}{17}e^{-t/2}\sin(t)$$

6.5#5

$y'' + 2y' + 3y = \sin(t) + \delta(t-3\pi)$ $y(0) = 0, y'(0) = 0$

We take Transform on both sides. Left hand side is

$$(s^2 + 2s + 3)Y(s)$$

since the initial values are all zero.

Right hand side is

$$\frac{1}{s^2+1} + e^{-3\pi s}$$

Hence, we have

$$Y(s) = \frac{1}{(s^2+1)((s+1)^2+2)} + \frac{1}{(s+1)^2+2}e^{-3\pi s}$$

The second term is easy:

$$\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{(s+1)^2+2}e^{-3\pi s} \to \frac{1}{\sqrt{2}}u_{3\pi}e^{-(t-3\pi)}\sin(\sqrt{2}(t-3\pi))$$

For the first term, we have to do the partial fractions

$$\frac{1}{(s^2+1)((s+1)^2+2)} = \frac{A_1 s + B_1}{s^2+1} + \frac{A_2 s + B_2}{(s+1)^2+2}$$

$$A_1 + A_2 = 0$$
$$2A_1 + B_1 + B_2 = 0$$
$$3A_1 + 2B_1 + A_2 = 0$$
$$3B_1 + B_2 = 1$$

We solve

$$A_2 = B_1 = B_2 = 1/4, A_1 = -1/4$$

Hence, we have

$$-\frac{1}{4}\frac{s}{s^2+1} + \frac{1}{4}\frac{1}{s^2+1} + \frac{1}{4}\frac{s+1}{(s+1)^2+2} \to -\frac{1}{4}\cos(t) + \frac{1}{4}\sin(t) + \frac{1}{4}e^{-t}\cos(\sqrt{2}t)$$

The solution is therefore

$$y(t) = -\frac{1}{4}\cos(t) + \frac{1}{4}\sin(t) + \frac{1}{4}e^{-t}\cos(\sqrt{2}t) + \frac{1}{\sqrt{2}}u_{3\pi}e^{-(t-3\pi)}\sin(\sqrt{2}(t-3\pi))$$

6.5#9

$y'' + y = u_{\pi/2} + 3\delta(t - 3\pi/2) - u_{2\pi}$. $y(0) = y'(0) = 0$

The transform of the left hand side is

$$(s^2+1)Y(s)$$

since the initial conditions are zero.

The transform of the right hand side is

$$e^{-s\pi/2}\frac{1}{s} + 3e^{-s3\pi/2} - e^{-s2\pi}\frac{1}{s}$$

We hence have

$$Y(s) = \frac{1}{s(s^2+1)}[e^{-\frac{\pi}{2}s} - e^{-2\pi s}] + 3\frac{1}{s^2+1}e^{-\frac{3\pi}{2}s}$$

We have

$$y(t) = u_{\pi/2}h(t-\pi/2) - u_{2\pi}h(t-2\pi) + 3u_{3\pi/2}\sin(t-3\pi/2)$$

where h satisfies

$$\mathcal{L}(h) = \frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

We determine that

$$A = 1, B = -1, C = 0$$

Hence, we have

$$h(t) = 1 - \cos(t)$$