1. \( y = y(t) \). Consider the equation

\[ y'' + 3y' + 4y = u_1(t), \quad y(0) = 1, y'(0) = 2 \]

where \( u_1(t) \) is the step function with the jump at \( t = 1 \). Find the Laplace Transform of \( y \), denoted as \( Y(s) \).

I was surprised to see that some people thought \( u_1(t) = u_1(t - 1) + u_1 \). This is not right. Here \( u_1(t) \) simply means the function \( u_1 \) and the variable is \( t \). If I want to mean multiplication, I should write \( tu_1 \) or \( u_1 \cdot t \).

We take the Transform on both sides.

\[
\mathcal{L}(lhs) = \mathcal{L}(y'') + 3\mathcal{L}(y') + 4\mathcal{L}(y) = [s^2Y - sy(0) - y'(0)] + 3[sY - y(0)] + 4Y = (s^2 + 3s + 4)Y - s - 5
\]

For the right hand side,

\[
\mathcal{L}(rhs) = \mathcal{L}(u_1) = e^{-s}\mathcal{L}(1) = \frac{e^{-s}}{s}
\]

Hence, \( Y(s) = \frac{e^{-s}}{s^2 + 3s + 4} \)

2. Given the Laplace Transform, find the function \( y(t) \):

\[
Y(s) = \frac{(s + 1)e^{-s}}{s^2 + 4s + 5}
\]

We first of simplify the expression so that we have the sin and cos:

\[
Y(s) = \frac{(s + 1)e^{-s}}{(s + 2)^2 + 1} = \frac{(s + 2)e^{-s}}{(s + 2)^2 + 1} - \frac{e^{-s}}{(s + 2)^2 + 1}
\]

Hence, we have

\[
y(t) = u_1e^{-2(t-1)}\cos(t - 1) - u_1e^{-2(t-1)}\sin(t - 1)
\]
1. $y = y(t)$. Consider the equation

$$y'' - y' + 2y = u_1(t), \quad y(0) = 2, y'(0) = 1$$

where $u_1(t)$ is the step function with the jump at $t = 1$. Find the Laplace Transform of $y$, denoted as $Y(s)$.

I was surprised to see that some people thought $u_1(t) = u_1(t - 1) + u_1$. This is not right. Here $u_1(t)$ simply means the function $u_1$ and the variable is $t$. If I want to mean multiplication, I should write $tu_1$ or $u_1 \cdot t$.

We take the Transform on both sides.

$$L(lhs) = L(y'') - L(y') + 2L(y) = [s^2Y - sy(0) - y'(0)] - [sY - y(0)] + 2Y$$

$$= (s^2 - s + 2)Y - 2s + 1$$

For the right hand side,

$$L(rhs) = L(u_1) = e^{-s}L(1) = \frac{e^{-s}}{s}$$

Hence, $Y = \frac{e^{-s}/s + 2s - 1}{s^2 - s + 2}$

2. Given the Laplace Transform, find the function $y(t)$:

$$Y(s) = \frac{(s - 1)e^{-s}}{s^2 - 4s + 5}$$

We first of simplify the expression so that we have the sin and cos:

$$Y(s) = \frac{(s - 1)e^{-s}}{(s - 2)^2 + 1} = \frac{(s - 2)e^{-s}}{(s - 2)^2 + 1} + \frac{e^{-s}}{(s - 2)^2 + 1}$$

Hence, we have

$$y(t) = u_1e^{2(t-1)}\cos(t - 1) + u_1e^{2(t-1)}\sin(t - 1)$$