Math 319 Quiz 3

Name: Section:

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1. $y = y(t)$. Consider the equation

$$y'' + 3y' + 4y = u_1(t), \quad y(0) = 1, y'(0) = 2$$

where $u_1(t)$ is the step function with the jump at $t = 1$. Find the Laplace Transform of y, denoted as $Y(s)$.

I was surprised to see that some people thought $u_1(t) = u_1(t-1) + u_1$. This is not right. Here $u_1(t)$ simply means the function u_1 and the variable is t. If I want to mean multiplication, I should write tu_1 or $u_1 \cdot t$.

We take the Transform on both sides.

$$\begin{aligned}\mathcal{L}(lhs) = \mathcal{L}(y'') + 3\mathcal{L}(y') + 4\mathcal{L}(y) &= [s^2Y - sy(0) - y'(0)] + 3[sY - y(0)] + 4Y \\ &= (s^2 + 3s + 4)Y - s - 5\end{aligned}$$

For the right hand side,

$$\mathcal{L}(rhs) = \mathcal{L}(u_1) = e^{-s}\mathcal{L}(1) = \frac{e^{-s}}{s}$$

Hence, $Y = \frac{e^{-s}/s+s+5}{s^2+3s+4}$

2. Given the Laplace Transform, find the function $y(t)$:

$$Y(s) = \frac{(s+1)e^{-s}}{s^2 + 4s + 5}$$

We first of simplify the expression so that we have the sin and cos:

$$Y(s) = \frac{(s+1)e^{-s}}{(s+2)^2 + 1} = \frac{(s+2)e^{-s}}{(s+2)^2 + 1} - \frac{e^{-s}}{(s+2)^2 + 1}$$

Hence, we have

$$y(t) = u_1 e^{-2(t-1)} \cos(t-1) - u_1 e^{-2(t-1)} \sin(t-1)$$

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1. $y = y(t)$. Consider the equation

$$y'' - y' + 2y = u_1(t), \quad y(0) = 2, y'(0) = 1$$

where $u_1(t)$ is the step function with the jump at $t = 1$. Find the Laplace Transform of y, denoted as $Y(s)$.

I was surprised to see that some people thought $u_1(t) = u_1(t-1) + u_1$. This is not right. Here $u_1(t)$ simply means the function u_1 and the variable is t. If I want to mean multiplication, I should write tu_1 or $u_1 \cdot t$.

We take the Transform on both sides.

$$\begin{aligned}\mathcal{L}(lhs) = \mathcal{L}(y'') - \mathcal{L}(y') + 2\mathcal{L}(y) &= [s^2Y - sy(0) - y'(0)] - [sY - y(0)] + 2Y \\ &= (s^2 - s + 2)Y - 2s + 1\end{aligned}$$

For the right hand side,

$$\mathcal{L}(rhs) = \mathcal{L}(u_1) = e^{-s}\mathcal{L}(1) = \frac{e^{-s}}{s}$$

Hence, $Y = \frac{e^{-s}/s + 2s - 1}{s^2 - s + 2}$

2. Given the Laplace Transform, find the function $y(t)$:

$$Y(s) = \frac{(s-1)e^{-s}}{s^2 - 4s + 5}$$

We first of simplify the expression so that we have the sin and cos:

$$Y(s) = \frac{(s-1)e^{-s}}{(s-2)^2 + 1} = \frac{(s-2)e^{-s}}{(s-2)^2 + 1} + \frac{e^{-s}}{(s-2)^2 + 1}$$

Hence, we have

$$y(t) = u_1 e^{2(t-1)} \cos(t-1) + u_1 e^{2(t-1)} \sin(t-1)$$