**Version 1:**
Consider the equation where $y = y(t)$:

$$2y^2 - y' + ty^2 = 0$$

Answer the questions below.

- (a). (5 pts) Find ALL solutions to the equation.
  
  Rearrange, $y' = y^2(t+2)$. The equation is separable. When $y^2 = 0$, we see $y = 0$. Hence there is a constant solution $y = 0$. For $y \neq 0$, we have

  \[
  \int \frac{dy}{y^2} = \int (t+2)dt \Rightarrow -\frac{1}{y} = \frac{1}{2}t^2 + 2t + C
  \]

  The solution could be $y = 0$ or $y = -1/(t^2/2 + 2t + C)$

- (b). (5 pts) Consider the solution curve that passes through $(0, 1)$ (in other words $y(0) = 1$). Determine the largest interval where this solution curve is defined. Find out where it attains the minimum value.

  For this solution curve, we determine $-1 = 0 + 0 + C$. Hence

  \[
y(t) = \frac{1}{1 - 2t - t^2/2}
  \]

  The solution is valid if and only if $1 - 2t - t^2/2 \neq 0$ or $t \neq -2 \pm \sqrt{6}$. Hence, the interval that contains 0 can be extended to $-2 - \sqrt{6}$ on the left and to $-2 + \sqrt{6}$ on the right. The largest interval is therefore $(-2 - \sqrt{6}, -2 + \sqrt{6})$

  The critical points can be found by setting $y'(t) = 0$. You can either do this by the equation or by the expression you get. If you do by the equation, you have $y' = 0 \Leftrightarrow y = 0$, or $t = -2$. However, $y = 0$ is impossible and the only choice is $-2$. If you do by the expression you get, you would have $(-\frac{1}{(t+2)^2})(t - 2) = 0$. Anyway, the only critical point is $t = -2$. Noticing that when $t$ approaches the boundary of the interval, the solution goes to infinity and thus there must be a minimum point in the interior. $t = -2$ is the only candidate and it must be the minimum point. $y_{min} = -\frac{1}{1-4-2} = \frac{1}{5}$

**Version 2:**
Consider the equation where \( y = y(t) \):

\[-(y + 1)^2 + y' = 2t(y^2 + 2y + 1)\]

Answer the questions below.

• (a). (5 pts) Find ALL solutions to the equation.

Rearrange, \( y' = (2t + 1)(y + 1)^2 \). The equation is separable. When \((y + 1)^2 = 0\), we see \( y = -1 \). Hence there is a constant solution \( y = -1 \). For \( y \neq -1 \), we have

\[
\int \frac{dy}{(y+1)^2} = \int (2t+1)dt \Rightarrow -\frac{1}{y+1} = t^2 + t + C
\]

The solution could be \( y = -1 \) or \( y = -1 - 1/(t^2 + t + C) \)

• (b). (5 pts) Consider the solution curve that passes through \((0,1)\) (in other words \( y(0) = 1 \)). Determine the largest interval where this solution curve is defined. Find out where it attains the minimum value.

For this solution curve, we determine \(-1/2 = 0 + 0 + C \). Hence

\[ y(t) = -1 - \frac{1}{t^2 + t - 1/2} \]

The solution is valid if and only if \( t^2 + t - 1/2 \neq 0 \) or \( t \neq (-1 \pm \sqrt{3})/2 \). Hence, the interval that contains 0 can be extended to \((-\sqrt{3} - 1)/2\) on the left and to \((\sqrt{3} - 1)/2\) on the right. The largest interval is therefore \(((-\sqrt{3} - 1)/2, (\sqrt{3} - 1)/2)\)

The critical points can be found by setting \( y'(t) = 0 \). You can either do this by the equation or by the expression you get. If you do by the equation, you have \( y' = 0 \Leftrightarrow y = -1, \ or \ t = -1/2 \). However, \( y = -1 \) is impossible and the only choice is \( -1/2 \). If you do by the expression you get, you would have \( \frac{1}{(2t + 1)^2} = 0 \). Anyway, the only critical point is \( t = -1/2 \). Noticing that when \( t \) approaches the boundary of the interval, the solution goes to infinity and thus there must be a minimum point in the interior. \( t = -1/2 \) is the only candidate and it must be the minimum point. \( y_{\text{min}} = y(-1/2) = 1/3 \)