Version 1:

Consider the equation where $y = y(t)$:

$$2y^2 - y' + ty^2 = 0$$

Answer the questions below.

- (a). (5 pts) Find **ALL** solutions to the equation.

  Rearrange, $y' = y^2(t+2)$. The equation is separable. When $y^2 = 0$, we see $y = 0$. Hence there is a constant solution $y = 0$. For $y \neq 0$, we have

  $$\int \frac{dy}{y^2} = \int (t+2)dt \Rightarrow -\frac{1}{y} = \frac{1}{2}t^2 + 2t + C$$

  The solution could be $y = 0$ or $y = -1/(t^2/2 + 2t + C)$

- (b). (5 pts) Consider the solution curve that passes through $(0, 1)$(in other words $y(0) = 1$). Determine the largest interval where **this** solution curve is defined. Find out where it attains the minimum value.

  For this solution curve, we determine $-1 = 0 + 0 + C$. Hence

  $$y(t) = \frac{1}{1 - 2t - t^2/2}$$

  The solution is valid if and only if $1 - 2t - t^2/2 \neq 0$ or $t \neq -2 \pm \sqrt{6}$. Hence, the interval that contains 0 can be extended to $-2 - \sqrt{6}$ on the left and to $-2 + \sqrt{6}$ on the right. The largest interval is therefore $(-2 - \sqrt{6}, -2 + \sqrt{6})$

  The critical points can be found by setting $y'(t) = 0$. You can either do this by the equation or by the expression you get. If you do by the equation, you have $y' = 0 \Leftrightarrow y = 0, \quad or\ t = -2$. However, $y = 0$ is impossible and the only choice is -2. If you do by the expression you get, you would have $(-\frac{1}{(...)^2})(-t-2) = 0$. Anyway, the only critical point is $t = -2$. Noticing that when t approaches the boundary of the interval, the solution goes to infinity and thus there must be a minimum point in the interior. $t = -2$ is the only candidate and it must be the minimum point. $y_{min} = -\frac{1}{1-4-2} = \frac{1}{5}$

Version 2:

Consider the equation where $y = y(t)$:

$$-(y+1)^2 + y' = 2t(y^2 + 2y + 1)$$

Answer the questions below.

- (a). (5 pts) Find **ALL** solutions to the equation.

  Rearrange, $y' = (2t+1)(y+1)^2$. The equation is separable. When $(y+1)^2 = 0$, we see $y = -1$. Hence there is a constant solution $y = -1$. For $y \neq -1$, we have

  $$\int \frac{dy}{(y+1)^2} = \int (2t+1)dt \Rightarrow -\frac{1}{y+1} = t^2 + t + C$$

  The solution could be $y = -1$ or $y = -1 - 1/(t^2 + t + C)$

- (b). (5 pts) Consider the solution curve that passes through $(0, 1)$(in other words $y(0) = 1$). Determine the largest interval where **this** solution curve is defined. Find out where it attains the minimum value.

  For this solution curve, we determine $-1/2 = 0 + 0 + C$. Hence

  $$y(t) = -1 - \frac{1}{t^2 + t - 1/2}$$

  The solution is valid if and only if $t^2 + t - 1/2 \neq 0$ or $t \neq (-1 \pm \sqrt{3})/2$. Hence, the interval that contains 0 can be extended to $(-\sqrt{3} - 1)/2$ on the left and to $(\sqrt{3} - 1)/2$ on the right. The largest interval is therefore $((-\sqrt{3} - 1)/2, (\sqrt{3} - 1)/2)$

  The critical points can be found by setting $y'(t) = 0$. You can either do this by the equation or by the expression you get. If you do by the equation, you have $y' = 0 \Leftrightarrow y = -1, \quad or\ t = -1/2$. However, $y = -1$ is impossible and the only choice is $-1/2$. If you do by the expression you get, you would have $\frac{1}{(...)^2}(2t+1) = 0$. Anyway, the only critical point is $t = -1/2$. Noticing that when t approaches the boundary of the interval, the solution goes to infinity and thus there must be a minimum point in the interior. $t = -1/2$ is the only candidate and it must be the minimum point. $y_{min} = y(-1/2) = 1/3$