#1.

The equation is

$$(2t-y)+(2y-t)y'=0$$

We check that $M_y=-1=N_t$. Hence, it's exact.

$$(2t-y)dt+(2y-t)dy=0$$

Undoing the differentiation, we have

$$d(t^2)-d(ty)+d(y^2)=0\Rightarrow d(t^2-yt+y^2)=0$$

Hence, we have

$$t^2-yt+y^2=C$$

Using the initial value, we have $C=13$.

To determine the interval, you can either solve it exactly as

$$y=\frac{t\pm\sqrt{t^2-4(t^2-13)}}{2}$$

Clearly, we pick the '+' branch and hence

$$y=\frac{t+\sqrt{52-3t^2}}{2}$$

from which we know $52-3t^2>0$ or $t^2<52/3$ and the interval is $(-\sqrt{52/3},\sqrt{52/3})$.

Another way is to notice $y'=\infty$ if $2y-t=0$ or $y=t/2$. Plugging this in, we have

$$t^2-t*t/2+(t/2)^2=13$$

Again $t^2=52/3$. Anyway, the answer is

$$t^2-yt+y^2-13=0, t\in(-\sqrt{52/3},\sqrt{52/3})$$

#2.

I have gone over a very similar example in discussions actually

$y+(2t-3ye^y)y'=0$ This is nonlinear, not separable and not exact. However, let $M=y, N=2t-3ye^y$, we see that $(M_y-N_t)/(-M)$ depends on y only and hence there's a factor that depends on y only. Then,

$$(\mu M)_y=(\mu N)_t$$

implies

$$\mu_y=\frac{1}{y}\mu$$

and thus $\mu = y$.

Hence, $y^2 dt + (2ty - 3y^2 e^y)dy = 0$. Undoing the differentiation, we have

$$d(ty^2) + (-3y^2 e^y)dy = 0$$

For the second term, we observe that

$$\int e^y y^2 dy = y^2 e^y - 2ye^y + 2e^y + C$$

Hence, the equation can be reduced to

$$d(ty^2) + d(-3y^2 e^y + 6ye^y - 6e^y) = 0$$

or

$$ty^2 - 3y^2 e^y + 6ye^y - 6e^y = C$$