The problem I was talking about in 343 this morning is as following:

\[ y + (2t + y)y' = 0 \]
\[ y(0) = \sqrt[3]{4} \]

This is a little different from the one in 341 because I thought the one in 341 was not typical enough.

It’s not hard to see \((M_y - N_t)/(-M) = 1/y\) only depends on \(y\) and hence there is an integrating factor of the form \(\mu = \mu(y)\) and

\[ \mu_y = \frac{1}{y} \mu \]

The factor is \(y\). Multiplying the factor and undoing the differentiation, we find

\[ d(y^3/3 + y^2t) = 0 \]

With the initial condition, we determine that \(y^3/3 + y^2t = 4/3\)

Then, I was talking about how to find the interval of definition. I told you that either \(y\) or \(y'\) blows up at the boundary. By the equation

\[ y' = \frac{-y}{2t + y} \]

We have if \(y = -2t\), then \(y' = \infty\). Plugging this into the curve, we have \(t^3 = 1\) and \(t = 1\). I then claimed that \(t = 1\) is the boundary and the interval is \((\infty, 1)\). Some students pointed out that the curve is fine at \(t = 1\) and showed me the curve. I was stuck there.

Now I check that actually at \(t = 1\), \(y > 0\) for the branch we are solving, and the condition we start with \(y = -2t\) is not satisfied. Hence it’s OK there. The interval is then \((-\infty, \infty)\) since \(y\) is solvable for any \(t\) and thus \(y\) doesn’t blow up.

If we want to make the problem interesting, we should change it into

\[ y + (2t + y)y' = 0 \]
\[ y(109/9) = -1/3 \]

to pick out the lower branch and the interval now is \((1, \infty)\).