The problem I was talking about in 343 this morning is as following:

$$y + (2t + y)y' = 0$$
$$y(0) = \sqrt[3]{4}$$

This is a little different from the one in 341 because I thought the one in 341 was not typical enough.

It's not hard to see $(M_y - N_t)/(-M) = 1/y$ only depends on y and hence there is an integrating factor of the form $\mu = \mu(y)$ and

$$\mu_y = \frac{1}{y}\mu$$

The factor is y. Multiplying the factor and undoing the differentiation, we find

$$d(y^3/3 + y^2 t) = 0$$

With the initial condition, we determine that $y^3/3 + y^2 t = 4/3$

Then, I was talking about how to find the interval of definition. I told you that either y or y' blows up at the boundary. By the equation

$$y' = \frac{-y}{2t + y}$$

We have if $y = -2t$, then $y' = \infty$. Plugging this into the curve, we have $t^3 = 1$ and $t = 1$. I then claimed that $t = 1$ is the boundary and the interval is $(-\infty, 1)$. Some students pointed out that the curve is fine at $t = 1$ and showed me the curve. I was stuck there.

Now I check that actually at $t = 1$, $y > 0$ for the branch we are solving, and the condition we start with $y = -2t$ is not satisfied. Hence it's OK there. The interval is then $(-\infty, \infty)$ since y is solvable for any t and thus y doesn't blow up

If we want to make the problem interesting, we should change it into

$$y + (2t + y)y' = 0$$
$$y(109/9) = -1/3$$

to pick out the lower branch and the interval now is $(1, \infty)$.