1. Consider \( f(x, y) = \ln(1 + x^2 2^y). \)
   
   (a). Compute the Taylor expansion of \( f \) about \((0, 0)\) up to second order.
   
   (b). Let \( g(x, y) = \ln(1 + x^2)2^y^3 \). Can you figure out \( g_{xx}(0, 0), g_{xy}(0, 0), g_{yy}(0, 0) \) without computing \( g_{xx}(x, y), g_{xy}(x, y), g_{yy}(x, y) \)? (Hint: No computation is needed. Use the result in (a) and do substitution.)

2. \( f(x, y) \) is differentiable (and hence continuous) and satisfies that \( f_x(x, y) = f_y(x, y) > 0 \) for all points (Caution: this doesn’t mean \( f_x = f_y \) is a constant). Inside the unit disk \( x^2 + y^2 \leq 1 \), does \( f \) have a global maximum and a global minimum? If yes, find them. (This is an old exam problem. For the boundary extremum, use Lagrange multiplier.)
(Bonus 2. 1 pt for each) Solve the integrals (I’m not expecting you to solve them all. Choose the ones you feel good to solve.)

a. \( \int \frac{\sqrt{y^2 - 49}}{y} \, dy \)

b. \( \int_0^1 \ln x \, dx \)

c. \( \int \frac{1}{(x+1)(x^2+1)} \, dx \)