234 Quiz 5

Section: *Name:*

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18 minutes. Bonus on back

1. (6) $F(x, y, z) = e^{x-y} + \ln z - z^2$. Consider the **zero set** of this function that passes through $(x_0, 1, 1)$.

   (a). Determine x_0 value and compute ∇F at this point

   (b). Compute the tangent plane of the zero set at point $(x_0, 1, 1)$.

2. (4) $f(x, y) = \ln(2+2x+e^y)$. Let's say C is the level set of f passing through $(1, 0)$. Locally around $(1, 0)$, could we regard the level set C as the graph of an implicit function $y = g(x)$? If yes, compute $dy/dx|_{x=1} = g'(1)$.

(Bonus: 2 pts) Consider again $f(x,y) = \ln(2+2x+e^y)$ and the level set C that passes through $(1,0)$. Compute the tangent line of C at $(1,0)$ in two ways:

- Using $y = g(1) + g'(1) * (x-1)$
- Using the fact that ∇f is perpendicular with the tangent line and $\nabla f \cdot (\vec{x} - \vec{x}_0) = 0$

Verify that they agree.

Comment: This is true for $z = f(x,y)$ as well. The tangent plane computed using $z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$ should agree with the tangent plane for the level set $F(x,y,z) = f(x,y) - z = 0$