234 Quiz 3(Version 2)

Section: *Name:*

Consider the parabola $y = x^2$.

1. (6) One can use $x = t$ to parametrize the curve and get $\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$. The curvature κ could be computed using $\frac{1}{\|\vec{x}'(t)\|^3}\|\vec{x}'(t) \times \vec{x}''(t)\|$. Find this curvature expression. (Notice that this formula only gives you the curvature but not the normal.)

2. (4) Set up the integral of the arclength for the portion between $(0, 0)$ and $(1, 1)$. You don't have to solve the integral but explain to me which technique of integration is suitable.

3. (Bonus:2) Suppose θ is the angle between the tangent and x-axis. The change of θ is given by $d\theta = \kappa ds = \kappa\|\vec{x}'(t)\|dt$. Compute the total change of the angle of the parabola $\int_{-\infty}^{\infty} \kappa\|\vec{x}'(t)\|dt$ and explain your answer.