234 Quiz 11

Section: *Name:*

20 minutes. Bonus on back.

1. (6) Let $\mathcal{C}$ be the boundary of the region bounded by $y = 0, x = 3, y = x$ and oriented counterclockwisely. Consider $\vec{v} = (y^2 - x^2, x^2 + y^2)$.

   (a). Compute the circulation $\oint_{\mathcal{C}} \vec{v} \cdot d\vec{x}$

   (b). Write out the formula for the outer flux of the field along the curve and change it into double integral without solving.

2. (4) Consider $\vec{F} = (e^y, xe^y)$ and $\mathcal{C}$: $y = \sqrt{x^5 + 1}$, $x : 0 \to 2$. Find $\int_{\mathcal{C}} \vec{F} \cdot d\vec{x}$

(Bonus) $\vec{v} = (P, Q) = (-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2})$ and $\mathcal{C}$ is the unit circle oriented counterclockwisely.

(a).(1.5) The circulation $\oint_{\mathcal{C}} \vec{v} \cdot d\vec{x} = 2\pi$. However, one is tempted to use Green's Theorem and get $\iint_D (-P_y + Q_x) dA$. $-P_y + Q_x = 0$. Does this example say the Green's Theorem is wrong, why?

(b).(1.5) You see that the circulation is nonzero, so it's not conservative in the whole domain where it's defined. Actually $\vec{v} = \nabla\theta$. Does this contradict with the fact that the gradient is always conservative?