234 Quiz 9

1. (a)(3) Mark true or false for the following(No need to explain):

   - $\int_a^b \int_c^d f(x,y)dxdy = \int_c^d \int_a^b f(x,y)dydx$ True. Rectangle. Fubini.
   - $\int_0^1 \int_{x^2}^x f(x,y)dydx = \int_{x^2}^x \int_0^1 f(x,y)dxdy$ False. Obviously the final answer can't depend on x. Not rectangle. Must change the limits for the iterated integral.
   - $\int_0^{100} \int_0^{100} f(x)f(y)dxdy = \left(\int_0^{100} f(x)dx\right)^2$ True.

   (b)(4) Let $D = \{(x,y) : 0 \le y \le 1, y^2 \le x \le 1\}$. Compute the volume under the graph of the function $f(x,y) = y\sin(x^2)$ and above D.

   The integral is
   $$\int_0^1 \int_{y^2}^1 y\sin(x^2)dxdy$$

   This integral is impossible to evaluate in this order. We change the order of integration. The region can be written as $\{(x,y) : 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$. Hence
   $$\int_0^1 \int_0^{\sqrt{x}} y\sin(x^2)dydx = \int_0^1 \frac{1}{2}x\sin(x^2)dx = -\frac{1}{4}\cos(x^2)|_0^1 = \frac{1}{4}(1-\cos 1)$$

2. (5) Set up the integral in polar coordinates without solving:

   The volume under $f(x,y) = x^2$ and above the region $D = \{(x,y) : x^2 + y^2 \ge 4, x^2 + (y-2)^2 \le 4\}$

   Soln. The integral is $\int_D f dA$.

   In polar coordinates, $x = r\cos\theta$ and hence $f = r^2\cos^2\theta$. The two regions in polar could be written as $r \ge 2$ and $r \le 4\sin\theta$ respectively. (For the second, plug in $x = r\cos\theta, y = r\sin\theta$, and you get $r^2 - 4r\sin\theta + 4 \le 4$).

   When the two curves intersect, you have $2 = r = 4\sin\theta$. Hence $\theta = \pi/6, 5\pi/6$. The region is therefore $\pi/6 \le \theta \le 5\pi/6, 2 \le r \le 4\sin\theta$.

   $dA = rdrd\theta$ and the integral is
   $$\int_{\pi/6}^{5\pi/6} \int_2^{4\sin\theta} r^2\cos^2\theta(rdrd\theta) = \int_{\pi/6}^{5\pi/6} \int_2^{4\sin\theta} r^3\cos^2\theta drd\theta$$