1. (a)(3) Mark true or false for the following (No need to explain):

- \[ \int_a^b \int_c^d f(x,y) \, dx \, dy = \int_c^d \int_a^b f(x,y) \, dy \, dx \] True. Rectangle. Fubini.
- \[ \int_A^B \int_C^D f(x,y) \, dy \, dx = \int_C^D \int_A^B f(x,y) \, dx \, dy \] False. Obviously the final answer can’t depend on \( x \). Not rectangle. Must change the limits for the iterated integral.
- \[ \int_0^{100} \int_0^{100} f(x,y) \, dx \, dy = \left( \int_0^{100} f(x) \, dx \right)^2 \] True.

(b)(4) Let \( D = \{(x,y) : 0 \leq y \leq 1, y^2 \leq x \leq 1\} \). Compute the volume under the graph of the function \( f(x,y) = y \sin(x^2) \) and above \( D \).

The integral is
\[ \int_0^1 \int_{y^2}^1 y \sin(x^2) \, dx \, dy \]

This integral is impossible to evaluate in this order. We change the order of integration. The region can be written as \( \{(x,y) : 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}\} \). Hence
\[ \int_0^1 \int_0^{\sqrt{x}} y \sin(x^2) \, dy \, dx = \int_0^1 \frac{1}{2} x \sin(x^2) \, dx = \frac{1}{4} \cos(x^2)|_0^1 = \frac{1}{4}(1 - \cos 1) \]

2. (5) Set up the integral in polar coordinates without solving:

The volume under \( f(x,y) = x^2 \) and above the region \( D = \{(x,y) : x^2 + y^2 \geq 4, x^2 + (y - 2)^2 \leq 4\} \)

Soln. The integral is \( \int_D f \, dA \).

In polar coordinates, \( x = r \cos \theta \) and hence \( f = r^2 \cos^2 \theta \). The two regions in polar could be written as \( r \geq 2 \) and \( r \leq 4 \sin \theta \) respectively. (For the second, plug in \( x = r \cos \theta, y = r \sin \theta \), and you get \( r^2 - 4r \sin \theta + 4 \leq 4 \)).

When the two curves intersect, you have \( 2 = r = 4 \sin \theta \). Hence \( \theta = \pi/6, 5\pi/6 \). The region is therefore \( \pi/6 \leq \theta \leq 5\pi/6, 2 \leq r \leq 4 \sin \theta \).

\( dA = r \, dr \, d\theta \) and the integral is
\[ \int_{\pi/6}^{5\pi/6} \int_2^{4 \sin \theta} r^2 \cos^2 \theta (r \, dr \, d\theta) = \int_{\pi/6}^{5\pi/6} \int_2^{4 \sin \theta} r^3 \cos^2 \theta \, dr \, d\theta \]