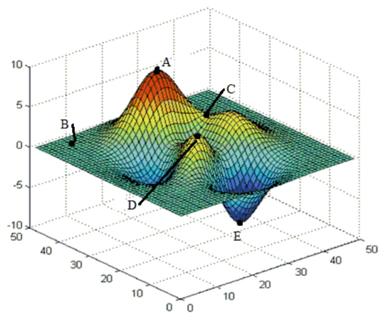
Keys-Quiz 6

1. (7) (a). In the figure below, $B(8, 42, 0)$ and for all $(x, y), x \leq 8, y \geq 42$, the function value is 0(namely the function is all zero on the left-upper corner of B). For points A, B, C, D, E, classify them into local maxima, local minima and saddle points(Hint: a global maximum is definitely a local maximum).



Soln. A is a global max and thus also a local max; C is a saddle point(in one direction, it's min but in another direction, it's max. In all, it's a saddle.); D is a local max and E is a local min. For B, notice that in a small ball that contains B, the function value is at least 0 which is the value at B. Thus B is a local min.

(b). Suppose the gradient of f exists everywhere in the domain. Mark true or false for the following. No need to explain.

- If $\nabla f(a, b) = 0$, (a, b) is either a local maximum or a local minimum. False. Consider a saddle.
- If f achieves the global maximum at (a, b), $f_x(a, b) = 0, f_y(a, b) = 0$. False. Consider a boundary maximum.
- If f achieves the global minimum at (a, b) which is an interior point, then $\nabla f(a, b) = 0$. True. You should compare with the previous one.
- If (a, b) is an interior local maximum and $(x(t), y(t))$ is a parametrized smooth curve so that $x(1) = a, y(1) = b$, then $g(t) = f(x(t), y(t))$ has a local maximum at $t = 1$ and $g'(1) = 0$. True.

2. (3) Find all critical points for $f(x, y) = xye^{-x-y}$.

Soln. $\nabla f = \vec{0}$. Then, $f_x = 0, f_y = 0$.

$$f_x = (xy)_x e^{-x-y} + xy(e^{-x-y})_x = ye^{-x-y} + xy(-e^{-x-y}) = ye^{-x-y}(1-x) = 0$$
$$f_y = xe^{-x-y}(1-y) = 0$$

By the first equation, $y = 0$ or $x = 1$. If $y = 0$, plugging into the second equation: $xe^{-x-0}(1-0) = 0$ or $x = 0$. If $x = 1$, plugging into the second equation yields $e^{-1-y}(1-y) = 0$ or $y = 1$. C.P.: $(0, 0)$ and $(1, 1)$