Keys-Quiz 4

1. (4) Consider the quadratic form $Q(x,y) = -2xy + \frac{1}{2}y^2$. Determine which kind of form this is and write it in standard form by completing the square. (Hint: if $x^2$ is present, you can use $x^2$ to complete the square, but $x^2$ is absent here and you can use $y^2$.)

Soln. In this quadratic form $A = 0, B = -2, C = 1/2$ and $4AC - B^2 = 0 - 4 < 0$. The form is indefinite.

To complete the square, we have

$$Q(x,y) = \frac{1}{2}(y^2 - 4xy) = \frac{1}{2}(y^2 - 4xy + 4x^2 - 4x^2) = \frac{1}{2}((y - 2x)^2 - 4x^2) = \frac{1}{2}(y - 2x)^2 - 2x^2 = u^2 - v^2$$

2. (6) Compute $f_x$ (or $\partial f/\partial x$) and $f_y$ (or $\partial f/\partial y$) for the two functions:

- $f(x,y) = xy \ln(xy)$
- $f(x,y) = g(\sin(xy))$

where in the second, $g$ is a single-variable function. (Notice that the first is your homework)

Soln. The first function is to test product rule.

$$f_x(x,y) = (xy)_x \ln(xy) + xy(\ln(xy))_x = y \ln(xy) + xy \frac{1}{xy}(xy)_x = y \ln(xy) + y$$

Similarly,

$$f_y(x,y) = x \ln(xy) + x$$

The second function is to test chain rule:

$$f_x(x,y) = g'(\sin(xy))(\sin(xy))_x = g'(\sin(xy)) \cos(xy)(xy)_x = g'(\sin(xy)) \cos(xy) y$$

Similarly,

$$f_y(x,y) = g'(\sin(xy)) \cos(xy) x$$

Bonus (2 pts) Consider the function $f(x,y) = \frac{xy}{x^2+y^2}$. If I define the value of this function at $(0,0)$ to be 0, would it be continuous at $(0,0)$?

Soln. Let $x = r \cos \theta, y = r \sin \theta$. Not at the origin, we have

$$f(x(r, \theta), y(r, \theta)) = \frac{\cos \theta \sin \theta}{\cos^2 \theta + \sin^2 \theta} = \cos \theta \sin \theta$$

The limit $(x,y) \to (0,0)$ is equivalent to $r \to 0$. We see that the limit depends on $\theta$ and the limit doesn’t exist.

The function can’t be continuous at the origin no matter what value we assign.