Keys-Quiz 4

1. (4) Consider the quadratic form $Q(x, y) = -2xy + \frac{1}{2}y^2$. Determine which kind of form this is and write it in standard form by completing the square.(Hint: if x^2 is present, you can use x^2 to complete the square, but x^2 is absent here and you can use y^2.)

   Soln. In this quadratic form $A = 0, B = -2, C = 1/2$ and $4AC - B^2 = 0 - 4 < 0$. The form is indefinite.

   To complete the square, we have

$$Q(x, y) = \frac{1}{2}(y^2 - 4xy) = \frac{1}{2}(y^2 - 4xy + 4x^2 - 4x^2)$$
$$= \frac{1}{2}((y - 2x)^2 - 4x^2) = \frac{1}{2}(y - 2x)^2 - 2x^2 = u^2 - v^2$$

2. (6) Compute f_x(or $\partial f/\partial x$) and f_y(or $\partial f/\partial y$) for the two functions:

   - $f(x, y) = xy \ln(xy)$
   - $f(x, y) = g(\sin(xy))$

   where in the second, g is a single-variable function. (Notice that the first is your homework)

   Soln. The first function is to test product rule.

$$f_x(x, y) = (xy)_x \ln(xy) + xy(\ln(xy))_x = y \ln(xy) + xy\frac{1}{xy}(xy)_x = y \ln(xy) + y$$

   Similarly.

$$f_y(x, y) = x \ln(xy) + x$$

   The second function is to test chain rule:

$$f_x(x, y) = g'(\sin(xy))(\sin(xy))_x = g'(\sin(xy)) \cos(xy)(xy)_x = g'(\sin(xy)) \cos(xy)y$$

   Similarly,

$$f_y(x, y) = g'(\sin(xy)) \cos(xy)x$$

Bonus(2 pts) Consider the function $f(x, y) = \frac{xy}{x^2+y^2}$. If I define the value of this function at $(0, 0)$ to be 0, would it be continuous at $(0, 0)$?

Soln. Let $x = r \cos\theta, y = r \sin\theta$. Not at the origin, we have

$$f(x(r, \theta), y(r, \theta)) = \frac{\cos\theta \sin\theta}{\cos^2\theta + \sin^2\theta} = \cos\theta \sin\theta$$

The limit $(x, y) \to (0, 0)$ is equivalent to $r \to 0$. We see that the limit depends on θ and the limit doesn't exist.

The function can't be continuous at the origin no matter what value we assign.