234 Quiz 2-Keys

1. (a). Suppose $\vec{a} = \begin{pmatrix} s \\ 1-s \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find the values of s, so that they make an acute angle. (Hint: $\cos\theta > 0$. Be sure to exclude $\theta = 0$ when $\vec{a} \parallel \vec{b}$.)

   Solution: Here, $\cos\theta > 0$ *implies* $\vec{a} \cdot \vec{b} > 0$ *if* $|\vec{a}| > 0, |\vec{b}| > 0$ *which are true in our case.*

   Therefore, $\vec{a} \cdot \vec{b} = s * 2 + (1-s) * 3 = 3 - s > 0$ *and* $s < 3$.

   However, if $\vec{a} \parallel \vec{b}$, *we have* $s/2 = (1-s)/3$*(or* $\vec{a} \times \vec{b} = 0$. *The 2D vectors could be augmented to 3D vectors by adding* 0 *in the last entry.) which implies that* $s = 2/5$ *if they are parallel.*

   The answer is $s < 3$ *and* $s \neq 2/5$.

   (b). For two vectors $\vec{a}, \vec{b}$, the value of $(\vec{a} \times \vec{b}) \cdot \vec{b}$ is _______. Why?

   Solution: The answer is 0 *because* $\vec{a} \times \vec{b} \perp \vec{b}$

2. (3) Suppose $A(1, -1, 2)$ and $B(2, 1, 3)$. Parametrize the **line segment** AB. (Hint: In other words, find a vector-valued function $\vec{x}(t)$ so that the curve it traces out as t varies is the line segment. If you like, you can think about a particle moving from A towards B with a constant velocity $\overrightarrow{AB}$.)

   Solution: If you start with A and go to B with some velocity $\vec{v}$, *the position you are at time* t *is* $\overrightarrow{OA} + t\vec{v}$. *Choosing* $\vec{v} = \overrightarrow{AB} = (1, 2, 1)$ *is convenient. Therefore, the line you get would be* $(1, -1, 2) + t(1, 2, 1)$. *For the line segment, you need* $0 \leq t \leq 1$.

Bonus: For a charged particle with charge q moving in a magnetic field $\vec{B}$, the Lorentz force acting on it is $\vec{F} = q\vec{v} \times \vec{B}$ where $\vec{v}$ is the velocity vector. Use Newton's law $\vec{F} = m\frac{d}{dt}\vec{v}$ to show that the speed $\|\vec{v}\|$ doesn't change if the Lorentz force is the only force acting on the particle. (2 pts)

Solution: It's clear that $\vec{F} \cdot \vec{v} = 0$ *and thus* $\vec{v} \cdot \frac{d}{dt}\vec{v} = 0$ *which means* $\frac{d}{dt}\|\vec{v}\|^2 = 0$.