Key-Quiz 1 (Version 2)

Given $A(0,0,1), B(2,1,3), C(-1,-1,0), D(-2,-4,5)$

1. Compute the distance from D to the plane ABC. (Hint: You can get a normal vector of ABC by computing $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$)

   Soln. First of all, let's find a normal vector of the plane: $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$. We have $\overrightarrow{AB} = (2-0, 1-0, 3-1) = (2,1,2)$ and $\overrightarrow{AC} = (-1,-1,-1)$.

   $\vec{n} = \hat{i}(1*(-1) - 2*(-1)) - \hat{j}(2*(-1) - 2*(-1)) + \hat{k}(2*(-1) - 1*(-1))$, or $\vec{n} = (1, 0, -1)$.

   A point on the plane is $A(0,0,1)$. $\overrightarrow{AD} = (-2,-4,4)$. The distance is

   $$d = \frac{|\vec{n} \cdot \overrightarrow{AD}|}{\|\vec{n}\|} = \frac{|-2-4|}{\sqrt{1^2+0+1^2}} = 3\sqrt{2}$$

2. There is a parallelepiped with AB, AC, AD to be some of its edges. Compute the volume of this parallelepiped.

   Soln. The volume is

   $$V = |(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}| = 6$$

Bonus: Suppose $|\vec{u}| = 2, |\vec{v}| = 1$. Suppose $|\vec{u} - \vec{v}| = 3/2$. Compute the angle between $\vec{u}$ and $\vec{v}$. (2 pts)

Soln. We see that $9/4 = |\vec{u} - \vec{v}|^2 = (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \|\vec{u}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$
We thus solve $\vec{u} \cdot \vec{v} = \frac{1}{2}(4 + 1 - 9/4) = 11/8$.
Hence $\cos\theta = (11/8)/(|\vec{u}||\vec{v}|) = 11/16$.
The angle is $\theta = \arccos(11/16)$