Keys-Quiz 12

1. Let $\mathcal{S}$ be the surface $xy - z = 2$ for $1 \leq x \leq 2, 1 \leq y \leq 2$. Set up the integral $\iint_{\mathcal{S}} \vec{v} \cdot \vec{N} dA$ for $\vec{v} = (x^2, zy, 1)$ without solving.

   First of all, the surface is the graph of $z = xy - 2$. We parametrize the surface as $\vec{x}(u, v) = (u, v, uv - 2)$ for $1 \leq u, v \leq 2$

   We have
$$\vec{N} dA = \vec{x}_u \times \vec{x}_v dudv = (-v, -u, 1) dudv$$
   and $\vec{v} = (u^2, (uv - 2)v, 1)$. The integral is thus
$$\int_1^2 \int_1^2 (-u^2 v - uv(uv - 2) + 1) dudv$$

2. $\vec{v} = (x, yz, xy)$. Let $\mathcal{S}$ be the sphere $x^2 + y^2 + z^2 = 4$. Write out the formula for the outer flux of $\vec{v}$ on $\mathcal{S}$ and change it into a volume integral in spherical coordinates.

   The formula is $\iint_{\mathcal{S}} \vec{v} \cdot \vec{N} dA$. Applying Divergence Theorem, we have
$$\iiint_R \nabla \cdot \vec{v} dV = \iiint_R (1 + z + 0) dV$$

   In spherical, we have $z = \rho \cos \phi, dV = \rho^2 \sin \phi d\rho d\phi d\theta$. The ranges: $0 \leq \rho \leq 2, 0 \leq \phi \leq \pi, 0 \leq \theta < 2\pi$. Hence, the integral is
$$\int_0^{2\pi} \int_0^{\pi} \int_0^2 (1 + \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta$$