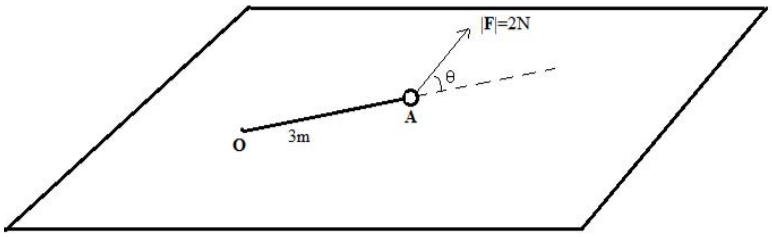
234 Quiz 1-Keys

1. As shown, a ball (A) is attached at the end of one string. The length of the string is $3m$. A force with size $2N$ is acting on the ball. $\theta = \pi/3$. Assuming O is picked as the reference point, we see that the position vector of A is $\vec{r} = \overrightarrow{OA}$. The torque acting on the ball is defined to be $\vec{T} = \vec{r} \times \vec{F}$. Draw the torque(roughly show the direction) in the figure and indicate the size of the torque(the magnitude of the torque vector).



Ans: By right hand rule, the torque, which is the cross product $\vec{r} \times \vec{F}$, is perpendicular with the plane and points up. (I won't draw it here since it's not convenient.) The size is $\|\vec{T}\| = \|\vec{r}\|\|\vec{F}\| \sin(\pi/3) = 3\sqrt{3}(N \cdot m)$

2. Given three vectors

$$\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

compute the determinant $det(\vec{a}, \vec{b}, \vec{c}) = \vec{a} \cdot (\vec{b} \times \vec{c})$.

Ans:

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} -1 & 1 \\ 0 & 2 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 0 * |\dots| = -4 - 3 = -7$$

Bonus: Suppose $|\vec{u}| = 2, |\vec{v}| = 1$. The angle θ between $\vec{u}, \vec{v}$ satisfies $\cos\theta = 1/6$. Find the length of $2\vec{u} + 3\vec{v}$:

Ans:

$$|2\vec{u} + 3\vec{v}| = \sqrt{(2\vec{u} + 3\vec{v}) \cdot (2\vec{u} + 3\vec{v})} = \sqrt{4|\vec{u}|^2 + 12\vec{u} \cdot \vec{v} + 9|\vec{v}|^2}$$
$$= \sqrt{4 * 2^2 + 12 * |\vec{u}||\vec{v}| * \frac{1}{6} + 9} = \sqrt{29}$$