1. Let $\mathcal{C}$ be $y = \sin x$ from $(0,0)$ to $(\frac{\pi}{2}, y_1)$.

   (a). Set up the integral without solving: $\int_{\mathcal{C}} xy dy$

   $\vec{x}(t) = (t, \sin t), 0 \le t \le \pi/2.$

   $$\int_0^{\pi/2} t \sin t \cos t dt$$

   (b). Let $\vec{F} = (y, x)$. Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{x}$

   $$\int_0^{\pi/2} (\sin t + t \cos t) dt = \pi/2$$

2. Let $\mathcal{C}_1$ be the dogleg path from $(0,0)$ to $(1,1)$ through $(0,1)$, and $\mathcal{C}_2$ be the dogleg path from $(0,0)$ to $(1,1)$ through $(1,0)$.

   (a). Is $\vec{F} = (2xy, x^2)$ a conservative field? Evaluate $\int_{\mathcal{C}_1} \vec{F} \cdot d\vec{x}$. Is this equal to $\int_{\mathcal{C}_2} \vec{F} \cdot d\vec{x}$? (If it's conservative, use the fundamental theorem.)

   $P_y = Q_x$ yes. Then, $f = x^2 y + C$. The values of both are 1.

   (b). Let $\vec{F} = (x^2 y, 0)$. Answer the same questions.

   $P_y \ne Q_x$. No. The first integral is $\int_0^1 x^2 dx = 1/3$. The second is 0.

3. (a). Let $\mathcal{C}$ be the line segment from $(1,1,1)$ to $(2,2,2)$. Parametrize it and compute $d\vec{x}$ using your parametrization.

   $\vec{x}(t) = (1+t, 1+t, 1+t), 0 \le t \le 1$. $d\vec{x} = (1,1,1)dt$

   (b). Consider $\phi(x,y,z) = -D/r$ where $D$ is a constant and $r = \sqrt{x^2+y^2+z^2}$ is the distance from the origin. Compute $\vec{F} = -\nabla\phi$ using the chain rule $\phi_x = \phi_r r_x$ etc and the fact $r_x = x/\sqrt{x^2+y^2+z^2} = x/r$

   $-\nabla\phi = -D(x,y,z)/r^3$

   (c). Is $\vec{F}$ a conservative field in the first octant? Use the parametrization in (a) and the fundamental theorem to compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{x}$. Verify that the two methods agree.

   Yes. It's conservative since it's the gradient of something. For the integral, using fundamental theorem, it's just $-(\phi(2,2,2) - \phi(1,1,1))$. For the line integral using parametrization, check the quiz.

4. Exercises about flux integrals? Wait until I come back...