Review Working sheet 2

1. (Critical points, Taylor expansion, and 2nd derivative test)

   (a) Find the Taylor expansion of $f(x, y) = 2^{x-y^2}$ at $(1, 1)$ up to second order.

   (b) Find all critical points and apply the 2nd derivative test for the following:
   $$f(x, y) = 8x^4 + y^4 - xy^2$$

2. (Optimization with constraint, Lagrange multiplier)

   (a). Find all the points on the surface $xy - z^2 + 1 = 0$ that are closest to the origin.

   (b). Find the minimum of $f(x, y, z) = xyz$ under the constraint $x^2 + 2y^2 + z^2 = 1$.

   (c). $f(x, y, z) = (x-1)^2 + (y-2)^2 + z^2$. Find the largest and smallest value of f inside $x^2 + y^2 + z^2 \leq 16$.

   (d). Let $f(x, y) = x + y - xy$. Let D be the region bounded by $x = 0, y = 0, x + 2y = 4$. Find the maximum and minimum value of f on D.

3. (Double integral, iterated integral)

   (a). Compute $\iint_D \sqrt{x^3 + 1} dA$ where $D = \{(x, y) : 0 \leq y \leq 1, \sqrt{y} \leq x \leq 1\}$

   (b). Compute
   $$\int_0^1 \int_{y^2}^1 y(3x^2 + 1)^{1/3} dx dy$$

   (c). Evaluate $\iint_R \frac{y}{x^5+1} dA$ where R is the region bounded by $y = 0, y = x^2, x = 1$

4. (Double Integrals in polar coordinates)

   (a). Write the following curves and functions in polar coordinates:

   - $x^2 + (y-a)^2 = a^2$
   - $x^3 + xy^2 - y = 0$
   - $f(x, y) = 2x^2 + y^2 - x$

   (b). Find the volume of the region bounded by $z = x^2 + y^2$ and $z = y$.

   (c). Let $R = \{(x, y) : (x-1)^2 + y^2 \leq 1, x^2 + (y-1)^2 \leq 1\}$. Compute the volume of under $f(x, y) = x$ and above R.

   (d). Compute $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2) dx dy$

5. (Triple integrals)

   (a). Evaluate $\iiint e^{x+y+z} dV$ over the region $\{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq \ln y\}$.

   (b). Evaluate $\iiint_D xy dV$ where D is the region bounded by $y = x^2, x = y^2, z = 0, z = x + y$

   (c). Compute the integral of $f(x, y, z) = x$ over the region $0 \leq y \leq 1, x, z \geq 0, x + z \leq 2$

   (d). A cube has edge length 2 and density equals the square of the distance from one specific edge. Find the total mass and the center of mass.

6. (Triple integrals in cylindrical/spherical coordinates)

   (a). (hw #14)Evaluate $\iiint x^2 dV$ over the interior of the cylinder $x^2+y^2 = 1$ between $z = 0$ and $z = 5$.

   (b). (hw #19) Evaluate $\iiint \sqrt{x^2 + y^2} dV$ over the interior of $x^2+y^2+z^2 = 4$

   (c). (hw #17) Evaluate $\iiint yz dV$ over the region in the first octant inside $x^2 + y^2 - 2x = 0$ and under $x^2 + y^2 + z^2 = 4$.