Review Working sheet

1. (vectors. Dot/cross product)

   Suppose $A(0,0,1), B(2,1,3), C(-1,-1,0), D(-2,-4,5)$

   (a). Compute plane ABC

   (b). Distance from D to ABC

   (c). Angle between $\overrightarrow{AB}, \overrightarrow{AC}$

   (d). The volume of the parallelepiped that has edges AB, AC, AD

2. (Parametrized curve; This is from the exam of last year I was TAing for.)

   Consider

   $$\vec{x}(t) = \begin{pmatrix} t\sin t + \cos t \\ t\cos t - \sin t \\ t^2 \end{pmatrix}$$

   Compute the unit tangent, unit normal and curvature. Also compute the arclength from $t = 1$ to $t = 2$

3. (Quadratic form)

   $$Q(x, y) = x^2 + 6xy + 8y^2$$

   Which kind of form is this?

4. (Polar coordinates)

   (a). If $x < 0, y > 0$, find a formula for the polar angle θ. Use your formula to find r, θ for point $(-1, \sqrt{3})$

   (b). Sketch the surface $x^2 + y^2 - z^2 = 0$

5. (This is also an old exam; the purpose is to test partial derivatives. You can choose to use linear approximation)

   Compute the limit

   $$\lim_{h \to 0} \frac{f(x - h, y + h) - f(x, y)}{h}$$

6. (linear approximation, tangent of the graph, tangent of the level set)

   (a). Consider

   $$f(x, y) = xe^{\sin y}$$

   .

   - Compute the linear approximation at $(1, 0)$ and compute the value $1.02 * e^{\sin(0.01)}$ approximately;
   - compute the tangent plane of the graph at $(1, 0)$;
   - compute the tangent line of the level set that passes through $(1, 0)$.

(b). Consider

$$f(x, y, z) = xy^2z + (x + 4y + 2z)^3$$

.

- Compute the linear approximation at $(2, -1, 1)$;
- Compute the tangent hyperplane of the graph at $(2, -1, 1)$;
- compute the tangent plane of the level set that passes through $(2, -1, 1)$.

7. (Gradient; revised from homework)

   Consider $f(x, y) = x^2 + y^2 + z$.

   (a). Compute ∇f.

   (b). Find the fastest increasing direction and fastest decreasing direction at $(0, 0, 1)$.

   (c). Let's say the level set that passes through $(0, 0, 1)$ is C. Find a point on C where the normal of the level set is parallel with $(1, 1, 1)$.

8. (Chain rule)

   (a). Let $f(x, y) = \cos(xy) + y\cos(x^2)$. Suppose a bug is crawling along a curve $\vec{x}(t) = (t^2, t)$. The bug then feels the f as $f(x(t), y(t))$. Compute the changing rate that the bug feels: df/dt.

   (b). Let $f(x, y) = \cos(xy) + y\cos(x^2)$ and $x = s + t, y = s/t$. $z = g(s, t) = f(x(s, t), y(s, t))$. Compute $\partial z/\partial s = g_s$ and $\partial z/\partial t = g_t$.

   (c*). Consider the Cartesian point $(1, 1)$. If you know $\partial f/\partial r = 1$ and $\partial f/\partial \theta = 2$ when regarding f as a function of r, θ, could you compute $\partial f/\partial x$ when you regard f as a function of x, y?

9. (Implicit function)

   Compute $\partial z/\partial x$ at $(1, -1, 1)$ if $\ln(2x + yz) - xy - zx = 0$.

10. (Finding function from its partial derivatives; higher order derivatives)

    Suppose your region is a ball. Inside this ball, you have

$$P(x, y) = \frac{2x}{x^2 + 1} + 2xy^3 + y\cos(xy)$$
$$Q(x, y) = 3x^2y^2 + x\cos(xy) + e^y$$

    Is there a function f so that $f_x = P$ and $f_y = Q$? If yes, find all such functions and compute f_{xy}, f_{yy}