Math234 Quiz 1

1 Version 1

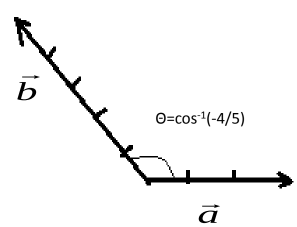
This is the one tested in lecture.

1. (a). Draw two vectors $\vec{a}, \vec{b}$ for which $\vec{a}$ has length 3 and $\vec{b}$ has length 5 and for which $\vec{a} \cdot \vec{b} = -12$.

   Solution:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta \Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{-12}{3 * 5} = -\frac{4}{5}$$

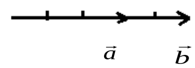
   This is between -1 and 0 and therefore the angle is obtuse. One possible drawing is:



   (b). The same as (a) except that $\vec{a} \cdot \vec{b} = 15$. Solution:

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta \Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{15}{3 * 5} = 1$$

   The angle between them is 0. Therefore, one plot would be:



Remark: I didn't go over this exact problem in discussions, but it shouldn't be difficult for you. If it was difficult, you really need to catch up.

2. $\vec{b}$ and $\vec{c}$ have the same sign and therefore we group them together:

$$\begin{aligned}
&(\vec{a} + (\vec{b} + \vec{c})) \times (-\vec{a} + (\vec{b} + \vec{c})) \\
&\qquad = -\vec{a} \times \vec{a} + \vec{a} \times (\vec{b} + \vec{c}) - (\vec{b} + \vec{c}) \times \vec{a} + (\vec{b} + \vec{c}) \times (\vec{b} + \vec{c}) \\
&\qquad = 2\vec{a} \times (\vec{b} + \vec{c})
\end{aligned}$$

I think this is simple enough. If you want, you can expand the parenthesis.

2 Version 2

This was not used.

1. Compute $(\vec{i} \times \vec{j}) \times \vec{i}$.

   Solution:

$$= \vec{k} \times \vec{i} = \vec{j}$$

or

$$= (\vec{i} \cdot \vec{i})\vec{j} - (\vec{j} \cdot \vec{i})\vec{i} = \vec{j}$$

2. (a). Find a normal vector for E_1 through $A(0,0,0)$, $C(1,1,0)$ $H(0,1,1)$

Solution: A normal vector can be picked as:

$$\vec{n} = \overrightarrow{AC} \times \overrightarrow{AH} =< 1,1,0 > \times < 0,1,1 >=< 1,-1,1 >$$

(b). The normal of another plane E_2 is $\vec{n}_2 =< 0,-1,2 >$. Find the angle between E_1 and E_2.

Solution: Assume the angle between the two planes is θ.

$$\cos\theta = |\frac{\vec{n} \cdot \vec{n}_2}{|\vec{n}||\vec{n}_2|}| = \frac{3}{\sqrt{3}\sqrt{5}} \Rightarrow \theta = \arccos(\frac{\sqrt{15}}{5})$$