Math234 Quiz 7

$z = f(x, y) = \sin x \sin y$.

Find all critical points and classify them.

Soln:

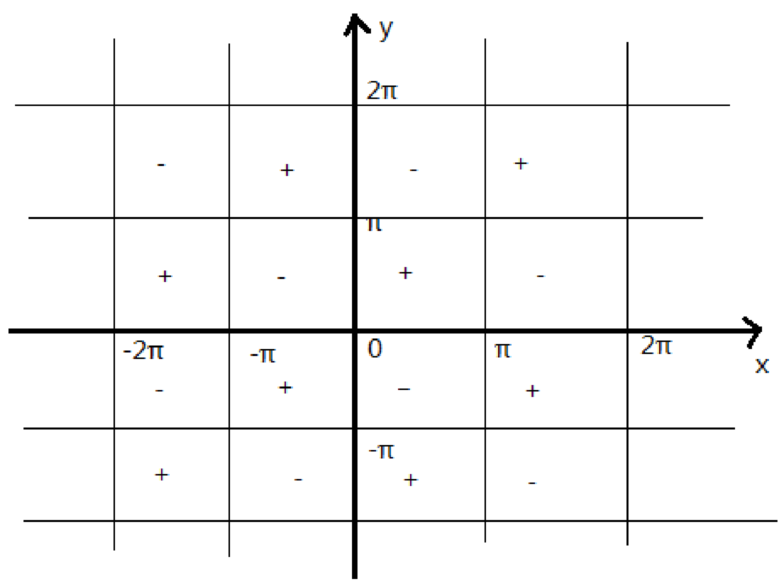
$$f_x = \cos x \sin y = 0 \Rightarrow x = \frac{\pi}{2} + m\pi \ or \ y = n\pi$$
$$f_y = \sin x \cos y = 0 \Rightarrow x = p\pi \ or \ y = q\pi + \frac{\pi}{2}$$

Above, m, n, p, q are all integers. (Many people got the zeros of $\cos x$ to be $k\pi/2$, which are not correct since this include the multiple of π where cos is nonzero.)

These two conditions therefore give the following critical points:

$$(\frac{\pi}{2} + m\pi, q\pi + \frac{\pi}{2})$$
$$(p\pi, n\pi)$$

To classify them, we need to look at the figure(as required by the problem):



Below, all letters mean some integers.

You can see that points $(p\pi, n\pi)$ are the grid points, so they are saddles.

The centers of positive squares will be maxima. They are $(\frac{\pi}{2}+m\pi, q\pi+\frac{\pi}{2})$ with both m and q to be odd or with both of them to be even. To be precise, they are: $(\frac{3\pi}{2}+2j\pi, \frac{3\pi}{2}+2k\pi)$ and $(\frac{\pi}{2}+2j\pi, \frac{\pi}{2}+2k\pi)$. At the first group, both $\sin x$ and $\sin y$ are -1 and at the second group, both $\sin x$ and $\sin y$ are 1.

The centers of negative squares will be minima. They are $(\frac{\pi}{2}+m\pi, q\pi+\frac{\pi}{2})$ with one integer to be odd and the other one to be even. To be precise, you can write as: $(\frac{\pi}{2}+2j\pi, \frac{3\pi}{2}+2k\pi)$ and $(\frac{3\pi}{2}+2j\pi, \frac{\pi}{2}+2k\pi)$. Here, one sine is 1 and the other sine is -1.