Math 222 Review

May, 2011

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1 Integration

1. Find the integrals below:

   1).
   $$\int \sqrt{1-x^2}dx$$

   2).
   $$\int \frac{x^3-2x+2}{x^2-4x}dx$$

2. Consider
   $$\int \frac{e^x}{e^{2x}-5e^x+6}dx$$

   1). Let's solve it together. Notice that this is not a fraction since it is not polynomial over polynomial. We do substitution $u = e^x$. Write out the new integral about u.

   2). Write out the partial fraction expression about u.

   3). Get the integral and then plug in $u = e^x$ back.

   4). Somebody may say $\frac{e^x}{(e^x-2)(e^x-3)} = \frac{-2}{e^x-2} + \frac{3}{e^x-3}$ and this is different from the partial fraction in 2). Is this equation wrong?

3. Consider $\int \ln x dx$:

   1). Calculate this indefinite integral.

   2). Calculate the limit by L'Hopital's principle:
   $$\lim_{x\to 0} x^m \ln x \quad m > 0$$

   3). Is
   $$I = \int_0^1 \ln x dx$$
   normal definite integral or improper integral? Get the number I

2 Series

1.

$$S_n = \sum_{k=1}^{n} a_k = \frac{n+1}{2n+1}$$

1). Does the corresponding series converge?

2). Find a_1 and a_n. Does this sequence of the nth term converge? Does the sequence of the nth partial sum converge?

2. Consider the series:

$$\sum_{n=2}^{\infty} (-1)^n 2^{2n+1} \frac{1}{5^n}$$

What kind of series is this? Find relevant quantities to get the sum.

3. Determine whether they converge:

a). $\sum \sin(\frac{n}{n+1})$. How about $\sum \ln(\frac{n}{n+1})$?

b). $\sum_{n=1}^{\infty} \frac{n^{3/2}}{n^3+4n}$

c). $\sum_{n=2}^{\infty} \frac{1}{n(1+(\ln n)^2)}$

d). $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n}$

4. a). Give me the Taylor expansion of $\frac{1}{1-x}$. Can you get the expansion of $\ln(1+x)$ from this?

b). Find the 10th derivative of

$$f(x) = \frac{x}{3-x}$$

at $x = 0$.

c). $f(x) = \sqrt{x+1}$. Get an estimation of $\sqrt{1.1}$ using Taylor polynomial such that the error is less than 0.01 and give your justification.

3 Complex numbers+ODEs

1. a). Simplify

$$\frac{1/2 + i\sqrt{3}/2}{e^{-i\pi/3}}$$

   b). Express $\sin(2\theta)$ and $\cos(2\theta)$ using $\sin\theta$ and $\cos\theta$

2. Solve the following:

   a).

$$y'(x) = e^{x-y}$$

   b).

$$xy' - y = 1, y(1) = 1$$

   c).

$$y'' + 2y' - 3y = x^2 + e^x, y(0) = 0, y'(0) = 1$$

4 Parametric curves

1. Consider the curve $x^3 + 2t^2 = 9, 2y^3 - 3t^2 = 4$.at $t = 2$

   1). Calculate the point (x_0, y_0) at $t = 2$.

   2). We want to calculate the slope at $t = 2$, but we don't want to calculate $x = x(t)$ first since this is complicated. Regard x and y as functions of t, differentiate these two equations directly with respect to t, and get the equations for $x'(2)$ and $y'(2)$ using the result in 1).

   3). Calculate the slope and the velocity vector at $t = 2$.

   4). Write out the tangent line at $t = 2$. Notice I only want parametric equation.

2. $x = t^2, y = t^3$

   1). Sketch this curve roughly.

   2). If $-1 \leq t \leq 1$, calculate the length corresponding to this portion.

   2). Calculate the area bounded by the portion of curve in 2) and the line $x = 1$.

3. $x = t^2 \cos(t), y = t^2 \sin(t)$

   1). Sketch the curve between 0 and 2π.

   2). Find the length between 0 and π

   3). If you regard t as the polar angle θ, what good things can you find?

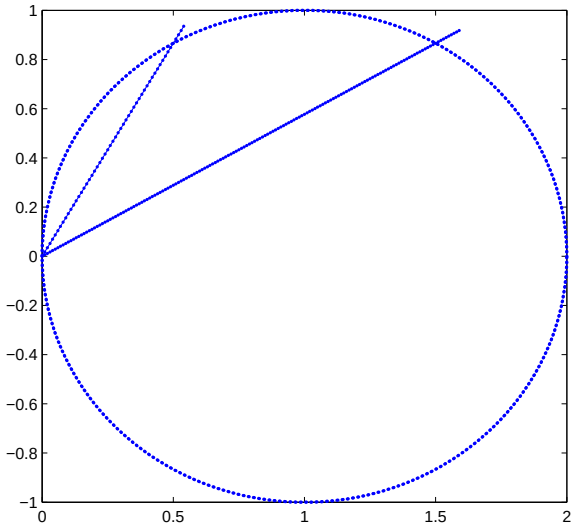
4. Consider the circle $x = 2 \sin t, y = 2 \cos t$

   a). Determine the direction of motion.

   b). Parameterize this curve with arc length parameter s and check $\int_2^5 \sqrt{x'(s)^2 + y'(s)^2} ds = 5 - 2 = 3$ directly.

5 Polar coordinates

1. Suppose we want to calculate the area bounded by those two lines and the circle in the figure.



   1). What kind of coordinate is more convenient for us to calculate the area? Write out the general formula for area.

   2). Find the center and radius of the circle. Write out the Cartesian equation.

   3). To get the polar equation, you just need to change the equation in 2) into polar equation. What's the answer?

   4). If I tell you the two lines are $y = \frac{\sqrt{3}}{3}x$ and $y = \sqrt{3}x$, give me the integral limits and the area.

2. Consider the four leaved rose $r = \cos(2\theta)$.

   a). Find the range of θ for the upper leaf.

   b). Find the length of that leaf.

   c). Find the area of that leaf.

   d). Calculate the tangent line at $\theta = 2\pi/3$

6 3D coordinate frame and vectors

1. What are the following in space:

   a). $x^2 + y^2 \leq 1$

   b). $x^2 + y^2 + (z-1)^2 = 4$, $x^2 + y^2 = 1$

   c). The set consisting of points such that for each point the sum of distances to $(1, 0, 0)$ and $(-1, 0, 0)$ is 4.

2. Giving $A(1, 2, 3), B(-1, 2, 4), C(0, 0, 3)$

   a). Find the angle $\angle CBA$

   b). Find the equation of line AB and line segment AB

   c). Find the distance between C and line AB and determine where the point on AB that can achieve this distance lies.

   d). Use cross product to calculate the area.

3. $\vec{u} = \hat{i} - \hat{j} + \hat{k}, \vec{v} = 2\hat{i} - \hat{k}, \vec{w} = 3\hat{j}$

   a). Find the volume of the parallelepiped determined by these three vectors.

   b). Find the length of $2\vec{u} + \vec{v}$

   c). Simplify $(2\vec{u} + \vec{v}) \times (\vec{u} - \vec{v})$ first and then calculate its value.

7 lines+planes

Look at the homework in 12.5. I wrote problems similar to them randomly.

1. Look at the three points in the previous section, and find the equation for that plane.
2. Given $P(2, -3, 4)$, find its distance to $x + 2y + 2z = 13$.
3. Find the line where $x - 2y + 4z = 2$ and $x + y - 2z = 5$ intersect.