Math 222 Quiz 6

March 9, 2011

Your Name: *Your Section:*

Instructions: You have 20 minutes to solve the following problems and the total score is 10 points. One bonus problem is on the back.

1. Solve the ordinary differential equation $\frac{dy}{dx} = e^{x-y}$ (3 pts)

2. $y' + (\tan x)y = \cos^2 x$
   a). Solve it. (3 pts) b). Check what you get in a) is the solution. (1 pt)

3. $xdy + x^4e^{-x}dx = 3ydx$(Hint: $y' = dy/dx$) (2+1 pts)
   a). If I tell you this is first order linear equation, get the standard form and solve it.
   b). If $y_1(x)$ is the solution satisfying $\lim_{x\to+\infty} y(x)$ exists, find $y_1(x)$ and get the limit.

(Bonus) In the picture, the electromotive force $U_0 = 1V$, the capacitance $C = 1F$ and the resistance $R = 1\Omega$. At first, the switch was on the left and there was no current. At $t = 0$, we turned the switch to the right.

1). It's known that the charge q on the capacitance and the voltage u_c satisfy $q = Cu_c$. We also know the current $i = \frac{dq}{dt}$. Ohm's Law: the voltage on the resistance is iR. Kirchhoff's law: $u_c + iR = 0$. The charge on the capacitance couldn't change immediately and thus the voltage wouldn't change at $t = 0$. Give out the differential equation that u_c satisfies and the initial condition $u_c(0)$. (2 pts)

2). Find the time when the voltage is $e^{-1}V$. (1 pt)