Math 222 Quiz 6
March 9, 2011

Instructions: You have 20 minutes to solve the following problems and the total score is 10 points. One bonus problem is on the back.

1. Solve the ordinary differential equation \( \frac{dy}{dx} = e^x - y \) (3 pts)

2. \( y' + (\tan x)y = \cos^2 x \)
   a). Solve it. (3 pts)  b). Check what you get in a) is the solution. (1 pt)

3. \( xdy + x^4 e^{-x} dx = 3ydx \) (Hint: \( y' = dy/dx \)) (2+1 pts)
   a). If I tell you this is first order linear equation, get the standard form and solve it.
   b). If \( y_1(x) \) is the solution satisfying \( \lim_{x \to +\infty} y(x) \) exists, find \( y_1(x) \) and get the limit.
(Bonus) In the picture, the electromotive force $U_0 = 1V$, the capacitance $C = 1F$ and the resistance $R = 1\Omega$. At first, the switch was on the left and there was no current. At $t = 0$, we turned the switch to the right.

1). It’s known that the charge $q$ on the capacitance and the voltage $u_c$ satisfy $q = Cu_c$. We also know the current $i = \frac{dq}{dt}$. Ohm’s Law: the voltage on the resistance is $iR$. Kirchhoff’s law: $u_c + iR = 0$. The charge on the capacitance couldn’t change immediately and thus the voltage wouldn’t change at $t = 0$. Give out the differential equation that $u_c$ satisfies and the initial condition $u_c(0)$. (2 pts)

2). Find the time when the voltage is $e^{-1}V$. (1 pt)