Math 222 Key to 3 in 17.3

By Lei March 28th, 2011

I made the units wrong in Sec. 367, because 'in' is not in the British System.
In one physics problem, if we use the units in the one fixed system, then everything will be consistent. Then we don't need to plug the symbols of these units into equations. In this problem, we use the British System. In this system, the unit of length is 'ft'. The unit of force is 'lb'. The unit of time is 's'. The unit of mass is 'slug' and the acceleration of gravity is $g = 32ft/s^2$. When I was in the department of physics in THU in China, we used the MKS System.

I can't draw the picture here and the picture is omitted.
In this problem, the original length of the spring is not important.
First of all, we put 20 lb and it stretches with 6 in. In 367, I thought in is in the British System, but it's not. Actually, 'ft' is the one in the system. Students in 368 told me that $1ft = 12in$, and thus $s_1 = 1/2ft$, which is the elongation of the spring. By Hook's, the force of the spring is ks_1. Since it's at equilibrium, we have $ks_1 = 20$ and thus $k = 40(lb/ft)$.
Then, we pulled $5in$ more and added $5lb$ more. The new position became $5/12ft$ below the **OLD** equilibrium position. However, since we added the new weight, the position was not $5/12ft$ below the **new** equilibrium position. The new equilibrium position could be figured out by $ks_2 = 25$ and thus $s_2 = 5/8ft$. The position at present was
$(6 + 5)/12 - 5/8 = 3.5/12ft$ or $3.5in$ below the new equilibrium position.

Denote the displacement from the equilibrium position as y and positive downward. We can see then $y(0) = 3.5/12$ instead of $5/12$.
The elongation of the spring is $s_2 + y$ and thus the force of the spring is $-k(s_2 + y)$ because downward is positive. The gravity is $25lb$. The resistance $-\delta y'$ is zero. The net force is $F = -k(s_2 + y) + 25 = -ky$. By Newton's law: $F = ma$, which is $-ky = my''$ and thus we have $my'' + ky = 0$.
Let's find m, $mg = gravity$ and thus $m * 32 = 25$. $g = 32ft/s^2$ in British System. (It's also $9.8m/s^2$). $m = 25/32slugs$. The equation becomes: $\frac{25}{32}y'' + 40y = 0$. $y(0) = 3.5/12$. The velocity is not of the unit in the system and we need to change 'in' into 'ft' and thus $y'(0) = v_0/12$.
The answer in the book is wrong because it uses the old equilibrium position!