1. a). $\int \frac{2y^2+y+1}{y^2+1}dy$ (3 pts)
    b). Does $\int_{1}^{\infty} \frac{\sqrt{x+1}}{x}dx$ converge? Why? (2 pts)

2. We already know $\int_{0}^{1} \ln xdx$. Now we want to find $\int_{0}^{1} x \ln xdx$ in homework:
   a). Is it improper? (1 pt) Calculate $\int_{0}^{1} x \ln xdx$ (2 pts)
   b). 2 pts if $\lim_{b \to 0^+} b^m (\ln b)^n$ where $m > 0, n > 0$ while 1.5 pts if $\lim_{b \to 0^+} b^2 \ln b$. 

Instructions: 20 minutes. Total score is 10. Bonus problems on the back.
Bonus 1: For rational functions, integral of \( \frac{1}{(ax^2 + bx + c)^n} \) \((a \neq 0, b^2 - 4ac < 0)\) isn’t covered. By substitution, it suffices to \( I_n = \int \frac{1}{(y^2 + 1)^n} dy \). By IBP \((u = \frac{1}{(y^2 + 1)^n}, dv = dy)\), we get
\[
I_n = \frac{n}{(y^2 + 1)^n} - \int y(-n) \left(\frac{2ny}{(y^2 + 1)^{n+1}}\right) dy = \frac{n}{(y^2 + 1)^n} + 2n \int \frac{y^2}{(y^2 + 1)^{n+1}} dy.
\]
Miracle is \( y^2 = (y^2 + 1) - 1 \).

a). Show me the miracle to find the relationship between \( I_n \) and \( I_{n+1} \) (1.5 pts)
b). Find the relationship between \( I_n \) and \( I_{n-1} \) (0.5 pt) and use this to find \( I_2 \) (1 pt)

Bonus 2: For what \( p \) does \( \int_1^\infty \frac{1}{x^p} dx \) converge? (0.5 pt) How about \( \int_0^\infty \frac{1}{x^p} dx \)? (1.5 pts)