Math 222 Warming up problems

September 2, 2010

Your Name: *Your Section:*

Instructions: These are some problems for review of Math 221 and warming up for lectures. They won't be graded. Please show your work.

1. Limits.

   (a) $\lim\limits_{x \to -5} \frac{x^2+3x-10}{x+5}$
   Factor the numerator, we get $(x-2)(x+5)$, and then we have:
   $\lim\limits_{x \to -5} \frac{x^2+3x-10}{x+5} = \lim\limits_{x \to -5} \frac{(x-2)(x+5)}{x+5} = \lim\limits_{x \to -5} (x-2) = -7$
   The problem itself is easy. What I want to emphasize here is that when we cancel $x+5$ from the numerator and denominator, we should ensure that it is not zero. Let's recall the definition of limit, it is NOT zero.

   (b) $\lim\limits_{x \to 4} \frac{4-x}{5-\sqrt{x^2+9}}$
   To get rid of the square root in the denominator, we multiply $5+\sqrt{x^2+9}$ on the top and the bottom. Then similarly, since x-4 is not zero, we have:
   $\lim\limits_{x \to 4} \frac{4-x}{5-\sqrt{x^2+9}} = \lim\limits_{x \to 4} \frac{(4-x)(5+\sqrt{x^2+9})}{(4+x)(4-x)} = \frac{5}{4}$

   (c) $\lim\limits_{x \to 1} \frac{\sin x - \sin 1}{\ln x}$
   Actually, I just want to remind of you the L'Hopital's principle here. Please have a review of the theorem. We have:
   $\lim\limits_{x \to 1} \frac{(\sin x)'|_{x=1}}{(\ln x)'|_{x=1}} = \cos 1$
   Another way is like this: since $x-1$ is not zero, we have $\lim\limits_{x \to 1} \frac{\frac{\sin x - \sin 1}{x-1}}{\frac{\ln x - \ln 1}{x-1}}$
   Qs: How about $\lim\limits_{x \to 1} \frac{\sin x}{\ln x}$? Does the L'Hopital's principle apply here?

   (d) $\lim\limits_{x \to 0} \frac{x \sin x}{2 - 2\cos x}$
   Applying L'Hopital's principle repeatedly, we can get the answer is 1.
   Some students in the section gave another good way: Multiplying $1+\cos x$ on the numerator and denominator.

2. Differentiating Implicitly.
   $y^2 = x^2 + \sin(xy)$. Find dy/dx as a function of x and y.
   Ans: $\frac{2x+y\cos(xy)}{2y-x\cos(xy)}$

3. Substitution Rule.
   $u = g(x)$ is differentiable, whose range is an interval I and f is continous on I. Then, we have:
   $\int f(g(x))g'(x)\mathrm{d}x = \int f(u)\mathrm{d}u$
   Example:
   $\int \frac{18\tan^2 x \sec^2 x}{(2+\tan^3 x)^2}\mathrm{d}x$
   Let $u = 2 + \tan^3 x$, we have $du = 3\tan^2 x \sec^2 x dx$. Thus we have $6\int \frac{1}{u^2}\mathrm{d}u$
   Ans:$-\frac{6}{2+\tan^3 x} + C$

4. Definition of Definite Integral. (Riemann)
   Does the function below have the Riemann Integral on [0,1]?
   f is defined on [0,1]. When x is rational, f(x) is 0.5, otherwise it's -0.5.
   Ans: No.
   Hint:Remember the integtal is the limit of the Riemann sum as the norm of the partion goes to 0. You can find some particular partions and points to let the Riemann sums have different limits.

5. Inverse Trigonometric Functions.

   (a) State the domains and codomains of arcsin, arccos, arctan, arccot, arcsec and arccsc.
   I'd like to remind you that when we define arcsec and arccsc, we use sec on $[0, \pi/2) \cup (\pi/2, \pi]$ and csc on $[-\pi/2, \pi/2] \setminus \{0\}$.

   (b) Derivatives of the inverse trigonometric functions.
   Using the method of implicit differentiation, we can get the results. Please refer to the textbook.
   Example: Find $\int_{-2/3}^{-\sqrt{2}/3} \frac{1}{y\sqrt{9y^2-1}}\mathrm{d}x$