Math 222 Quiz 4
September 30, 2010

Instructions: Time is 20 minutes and the total score is 10 points. You can answer in attached papers if you like. There is one extra problem on the back.

1. Choose ANY ONE of the two below. (3 points)
   - If \( \int_1^{+\infty} \frac{1}{x} \, dx \) means \( \lim_{b \to +\infty} \int_1^{b} \frac{1}{x} \, dx \), what does \( \int_0^{+\infty} \frac{1}{x} \, dx \) mean? Does it converge?
   - Why?
   - Find the error below and give out the correct answer:
     \[ \int_{-1}^{1} \frac{1}{x^{1/3}} \, dx = (-3x^{-1/3})|_{-1}^{1} = -3 + 3(-1)^{-1/3} = -6. \]

2. Calculation. (7 points)
   (a). \( \int_{0}^{\infty} ye^{-y} \, dy \) (3 pts)
   (b) i). Given 2, 7, 12, 17, 22, ... where each number except the first one minus its previous one is a constant, find the expression for the n-th term \( a_n \). (2 pts)
   ii). Find the limit \( \lim_{n \to \infty} \frac{a_n}{\sqrt{n^2 + 1}} \) (2 pts)
(Bonus) 2pts.
If \( a_1 = 1 \) and \( a_{n+1} = a_n / (n + 1) \), find \( \lim_{n \to \infty} 2^n a_n \)