Math 222 Review Problems

Nov. 11, 2010

Instructions: Time is 30 minutes. You can choose any group to join in or you can choose not to join any groups. For each group, try to solve as many problems in corresponding group as you can, and I have 3 cookies to give to the first three winners. You can discuss within groups, but I remind you that if you help others, you may lose your own cookie. I'll help you to decide who can get the cookies. Of course, if you don't join any group, you can't get cookie.

1. **Group One**

   1. Does this converge: $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n^2}$ and why?
   Ans: No, because the n-th term doesn't go to zero.
   2. Find the radius of convergence of the power series and for what x's does it converge?
   $\sum_{n=1}^{\infty}\frac{n!}{n^n}x^n$
   Ans: The radius is e and x's are $(-e, e)$ The ratio is $x\frac{n^n}{(n+1)^n} \to \frac{x}{e}$ When $x = e$ or $-e$, use $\lim_{n\to\infty}\frac{n!e^n}{n^n\sqrt{2\pi n}} = 1$
   3. Find the Maclaurin series: $f(x) = \frac{1}{(1-x)(2-x)}$
   Hint: Use the partial fraction first and then use the expansion for $1/(1-x)$
   $f(x) = \frac{1}{1-x} - \frac{1}{2-x} = \frac{1}{1-x} - \frac{1}{2}\frac{1}{1-x/2}$
   4. Find P_3 or T_3 for $f(x) = \sin x$, use it to calculate $\sin(50°)$ and estimate the error.
   Ans: $P_3 = x - x^3/3!$ and the error can be bounded by $(5\pi/18)^5/5!$
   5. Solve $(\cos x)y' - 2y\sin x = 2\sin(2x)$
   Ans: Divide by $\cos x$ first and get the integrating factor $\cos^2 x$
   $(\cos^2 xy)' = 4\sin x\cos^2 x$
   6. Solve $y^{(4)} - y'' - 2y = 0$
   Ans: Auxiliary equation $r^4 - r^2 - 2 = 0$ so answer should be
   $C_1e^{\sqrt{2}x} + C_2e^{-\sqrt{2}x} + C_3\cos x + C_4\sin x$
   7. For fun: We have a bag of rice, which is 1 lb. One day, a rat came and ate half of the rice. The second day, another rat came and ate 1/3 of what was left. The third, a rat came and ate 1/4 of what was left,and so on. On nth day, a rat came and ate $1/(n+1)$ of what was left. How much rice was left after infinity days?
   Ans: The n-th day, we the rat ate $1/n(n+1)$, so nothing was left.

2. **Group Two**

   1. Does this converge: $\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln(n^3)}$? Estimate the error if we only keep the first 10 terms.
   Ans: AST, so converges. The error is $1/(3\ln 12)$

2. Find the radius of convergence of the power series and for what x's does it converge?
$\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$
Ans: Convergence $[-1, 1)$. Use the ratio test. Note, $\ln n / \ln(n+1)$ goes to 1 by L'Hopital's Principle. $x = -1$,AST. $x = 1$, Integral Test.
3. Find the Maclaurin series: $f(x) = \ln(1 + x^3) - x\sin(x^2)$ and find $\lim_{x \to 0} \frac{f(x)}{x^6}$
Hint: Use the expansion for $\ln(1 + x)$ and $\sin x$
$\ln(x + 1) = x - x^2/2 + x^3/3 - x^4/4 + ...$, so $f(x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{3n}}{n} - \sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+3}}{(2n+1)!}$
4. Find the Taylor series for $x^3 - 3x^2 + 6x - 4$ at 1
Ans:$(x - 1)^3 + 3(x - 1)$
5. Solve $(1 - t^2)\frac{dz}{dt} - 1 - z^2 = 0$
Ans: Separable. $\int 1/(1 + z^2)dz = \int dt/(1 - t^2)$ The first is $\tan^{-1} z$ and the second can be solved by using partial fraction.
6. Solve $y'' + y' - 2y = 1 + x - 8e^{3x}$
Hint: Particular solution can be tried by $Ax + B + Ce^{3x}$.
7. For fun: We have a bag of rice, which is 1 lb. One day, a rat came and ate half of the rice. The second day, another rat came and ate 1/3 of what was left. The third, a rat came and ate 1/4 of what was left,and so on. On nth day, a rat came and ate $1/(n + 1)$ of what was left. How much rice was left after infinity days?

3. **Group Three**

1. Does this converge: $\sum_{n=1}^{\infty}(-1)^n \frac{\sin(n)}{n^2}$ and why?
Ans: Yes. Absolute convergence test.
2. Find the interval of convergence and get the sum
$\sum_{n=0}^{\infty}(\frac{x^2-1}{3})^n$
Ans: $(-2, 2)$ and the sum can be calculated by geometric series. First is 1 and ratio is $(x^2 - 1)/3$
3. Find the first 4 terms of the Maclaurin series: $f(x) = e^{-2x}(1 + 2x)$
$\sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} + \sum_{n=0}^{\infty} \frac{(-1)^n(2x)^{n+1}}{n!}$ and then write them out. I believe you can get the first four. It's said that you can stop at x^5
Hint: Use the expansion for e^x
4. Find Taylor series of $f(x) = e^x$ at 1 and estimate the error if we set $x = 2$ and use the first 4 terms to calculate e^2.
Omitted
5. Solve $\frac{dq}{dt} = (4 + q^2)\cos(2t)$
Hint: Separable. $\tan^{-1}(q/2) = \sin(2t) + C$
6. Solve $y'' - 5y' + 6y = e^{3x}$
Hint: Try the particular Axe^{3x}

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4. **Group Four**

1. Does this converge: $\sum_{n=2}^{\infty} \cos(n\pi)\frac{1}{1+\sqrt{n}}$? Estimate the error if we only keep the first 10 terms.
Ans: Yes, AST. Similar to Group 2
2. Find the radius of convergence of the power series and for what x's does it converge?
$\sum_{n=2}^{\infty}(n+1)x^{n-1}$
Ans: radius is 1 and $(-1, 1)$
3. Find the Maclaurin series: $f(x) = 2x\cos(x^2)$
Omitted.
4. Find P_4 or T_4 for $f(x) = \cos(\pi/3 + x)$, and estimate the error if $|x| < 0.1$.
I believe you can do it.
5. Solve $xy' + 3y = \sin x/x^2$
Ans: Divide by x first and then the integrating factor is x^3
6. Solve $y''' + 4y'' + 3y' = 0$
Ans: Two ways, the first is just to regard $u = y'$ and the second is to write out the auxiliary equation directly $r^3 + 4r^2 + 3r = 0$ $C_1 + C_2e^{-x} + C_3e^{-3x}$
7. For fun: We have a bag of rice, which is 1 lb. One day, a rat came and ate half of the rice. The second day, another rat came and ate 1/3 of what was left. The third, a rat came and ate 1/4 of what was left,and so on. On nth day, a rat came and ate $1/(n+1)$ of what was left. How much rice was left after infinity days?