Keys to Quiz8

By Lei November 4, 2010

1. Solve the differential equation with initial value $y(1) = 0$: $y' - \frac{y}{x} = x + 1$ (3 pts)
   Ans:First order, linear differential equation. It's of the standard form already.
   The coefficients are not continuous on R, but since I gave you $y(1) = 0$, you only need to consider $x > 0$.
   Itegrating factor $\mu(x) = e^{\int -1/x dx}$. We choose $\mu(x) = \frac{1}{x}$. Attention $e^{-\ln x} = (e^{\ln x})^{-1} = 1/x$, not $-x$!
   Then multiplying the integrating factor, we have $(\frac{y}{x})' = 1 + \frac{1}{x}$. Integrate, and we have $y = x(x + \ln x + C)$.
   $y(1) = 0$ implies $C = -1$. Hence, $y(x) = x^2 + x \ln x - x$

2. (a). Find the solution: $8y''(t) + 8y'(t) + 2y(t) = 0$ and $y(0) = 0$, $y'(0) = 1$ (4 pts)
   Ans:It's the equation with **constant** coefficients.
   The corresponding auxiliary equation is $8r^2 + 8r + 2 = 0$, which has only one root $r = -1/2$.
   The general solution $y(t) = C_1 e^{-t/2} + C_2 t e^{-t/2}$
   $y(0) = 0$ implies $C_1 = 0$ and then $y'(0) = 1$ implies $C_2 = 1$.
   Attention: The variable is t not x.
   (b). Find the general solution to $y'' + 2y' + 4y = 0$. y is a function of x(3 pts)
   Ans: This is your homework. The corresponding auxiliary equation is $r^2 + 2r + 4 = 0$ and the solution is $r_{1,2} = -1 \pm \sqrt{3}i$. Hence the general solution is
   $y(x) = e^{-x}(C_1 \cos(\sqrt{3}x) + C_2 \sin(\sqrt{3}x))$

   Bonus: Choose **ANY ONE** below: (4 pts)
   • a). If y_1 and y_2 are solutions to $y'' - 5y' + 6y = 0$, how about $y_1 + y_2$? (1 pt). If the equation is $y'' - 5y' + 6 = 0$, what's the answer? (1 pt)
   Ans:The first equation is linear and homogeneous, so $y_1 + y_2$ is the solution. The second is inhomogeneous, so $y_1 + y_2$ is not the solution. You can plug in to check.

   b). If I tell you that two solutions to the equation $x^2 y'' - 5xy' + 9y = 0$ are of the type $y_1 = x^r$ and $y_2 = x^r \ln x$, which are obviously linearly independent, find r (1 pt) and write out the general solution (1 pt)
   Ans:Attention please, the equation is **NOT** with **constant coefficients**, so it doesn't have the auxiliary equation. Since I told you that x^r and $x^r \ln x$ are both solutions. You can plug in any one to determine the r. Plug x^r in, and you'll have $r(r-1)x^r - 5rx^r + 9x^r = 0$ and $(r-3)^2 = 0$. Hence, $r = 3$. Since they are independent, you'll have the general solution as $y(x) = C_1 x^3 + C_2 x^3 \ln x$

   • Solve $2(yy')' - 10yy' + 6y^2 = 0$ (2 pts) and $y'' - 5y' + 6 = 0$ (2 pts).
   Hint: Both equations need substitution. For the second, you can regard y' as a whole

first and then solve y.
Ans: The first equation needs $u = y^2$ and you'll get $u'' - 5u' + 6u = 0$, which is easy to solve. The second needs $u = y'$ and you'll get $u' - 5u + 6 = 0$, which is also easy to solve. Then $y = \int u dx$. For the second, since you have learned the inhomogeneous equation, you can use the method you have learned to solve now.