Keys to Quiz6

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1. Find the radius of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\sqrt{n}3^n}$. For what values of x does the series converge? (4 pts)
   Ans: Using the ratio test, we can find that the ratio is $\frac{a_{n+1}}{a_n} = \frac{(x-1)n\sqrt{n}}{3(n+1)\sqrt{n+1}} \to \frac{x-1}{3}$. We then let $|\frac{(x-1)}{3}| < 1$ and we can get $|x-1| < 3$, which implies $R = 3$. $-2 < x < 4$.
   However, we must check the endpoints now. Plugging them in, we have serieses $\sum_{n=0}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$ and $\sum_{n=0}^{\infty} \frac{1}{n\sqrt{n}}$. Both converge absolutely because the sum of the absolute values are a convergent p-series with $p = 3/2 > 1$.
   Finally we have the x's are in the interval $[-2, 4]$

2. (a). Find the Taylor series generated by $f(x) = x^3 - 2x + 4$ at 2 (2 pts)
   Ans: This is the homework. We can find that $f'(x) = 3x^2 - 2$, $f''(x) = 6x$ and $f'''(x) = 6$. The higher orders are all zero. Hence the series is
   $8 + \frac{10}{1!}(x-2) + \frac{12}{2!}(x-2)^2 + \frac{6}{3!}(x-2)^3 = 8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$
   (b). Find the Maclaurin series expansion of $f(x) = \frac{1}{x^2+16}$ (4 pts)
   Hint: Use the expansion for $\frac{1}{x+1}$. Here you should factor 16 out and replace x with a suitable expression.
   Ans: Since $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$, we have
   $\frac{1}{x^2+16} = \frac{1}{16}\frac{1}{x^2/16+1} = \frac{1}{16}\sum_{n=0}^{\infty} (-1)^n (x^2/16)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}/16^{n+1}$. We are done.
   Question: Can you find out the radius of convergence for this series just by using the conclusion for the series of $1/(x+1)$ instead of calculating directly?