Keys to Quiz3

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a). $\int \frac{y^4+2y^2+y-81}{y(y^2+9)}\mathrm{d}y$ (Hint: Remember to check whether the fraction is improper or not.)(4 pts)
One way is $y^4+2y^2+y-81=(y^2+9)(y^2-9)+y(2y+1)$, and the other way is long division.
$y^4+2y^2+y-81=y(y^3+9y)-7y^2+y-81$, so $\frac{y^4+2y^2+y-81}{y(y^2+9)}=y+\frac{-7y^2+y-81}{y^3+9y}$ (Attention: **Not** $y-\frac{7y^2+y-81}{y^3+9y}$ **!**).
We finally have $\frac{y^4+2y^2+y-81}{y(y^2+9)}=y-\frac{9}{y}+\frac{2y+1}{y^2+9}$
Ans: $\frac{1}{2}y^2-9\ln|y|+\ln(y^2+9)+\frac{1}{3}\arctan(y/3)+C$
Note: Somebody doesn't know what to do with $\frac{2y+1}{y^2+9}$ and please get it.

b). Find the area of the region bounded by $y=\frac{x^2}{\sqrt{1+x^2}}$, $x=1$ and x-axis. (6 pts)
Sol:
$Area=\int_0^1 \frac{x^2}{\sqrt{x^2+1}}\mathrm{d}x$
Then use trig substitution $x=\tan\theta$, and you'll get $\int_0^{\pi/4}\frac{\tan^2\theta}{\sec\theta}\sec^2\theta\mathrm{d}\theta$
Note: Many students forgot $\sec^2\theta d\theta$ after the substitution, even though they knew $dx=\sec^2\theta d\theta$.
Then refer to your notes of the lectures on how to evaluate this integral.
Ans:$\frac{1}{2}(\sqrt{2}-\ln(\sqrt{2}+1))$