Keys to Quiz12

By Lei December 9, 2010

1. $A(1,2,3)$. Two planes $P1: x+y=1$ and $P2: 2x+y-2z=2$. Find the angle between planes $(2')$, the parametrizations of the line where the planes intersect $(2')$ and the distance between A and $P2$ $(1')$.

   Ans: Nomal vector of $P1$ is $n_1 =< 1,1,0 >$ and normal vector of $P2$ is $n_2 =< 2,1,-2 >$. The angle is thus $\theta = \cos^{-1}(|\frac{n_1 \cdot n_2}{|n_1||n_2|}|) = \cos^{-1}(\frac{1}{\sqrt{2}}) = \pi/4$.

   The line should be parallel to $n_1 \times n_2 =< -2,2,-1 >$. We need to find a point on the line. Any point satisfying $x+y=1$ and $2x+y-2z=2$ should work. I choose $P(1,0,0)$. The line is $x = 1-2t, y = 2t, z = -t$

   Distance $d = |\frac{\overrightarrow{AS} \cdot n_2}{|n_2|}|$. S is a point on $P2$. If we choose $S(1,0,0)$. We have $d = |\frac{<0,-2,-3> \cdot <2,1,-2>}{3}| = 4/3$

2. (a). $\mathbf{r}(t) = \cos 2t\mathbf{i} + 3\sin 2t\mathbf{j} + 4\mathbf{k}$. Find the velocity, the speed and acceleration at $t = \pi$ $(3')$
   (b). $\frac{d\mathbf{r}(t)}{dt} = \sec t \tan t\mathbf{i} + \tan t\mathbf{j} + 2\sin t\cos t\mathbf{k}$. $\mathbf{r}(0) = \overrightarrow{0}$. Find $\mathbf{r}(\pi/3)$ $(2')$

   Ans: (a). $v(t) = dr(t)/dt = -2\sin 2ti + 6\cos 2tj$ and $a(t) = dv(t)/dt = -4\cos 2ti - 12\sin 2tj$. At $t = \pi$, we have $v(\pi) = 6j$ and speed is $|6j| = 6$. $a(\pi) = -4i$

   (b). $r(\pi/3) = r(0) + \int_0^{\pi/3} \sec t \tan t\mathbf{i} + \tan t\mathbf{j} + 2\sin t\cos t\mathbf{k}dt = i + \ln 2j + 3k/4$. Here $\int \sec t \tan t dt = \sec t + C$, $\int \tan t dt = \ln|\sec t| + C$ and $2\sin t\cos t = \sin 2t$

   (Bonus) A particle is moving. If $\mathbf{v} \cdot d\mathbf{v}/dt = 1$ and the speed at $t = 0$ is 0, find the speed at $t = 2$ $(2')$
   Furthermore, let it be on $y^2 = 2x$. If at $t = 2$, it's at $(2,2)$ and moving from left to right, find the velocity $(2')$.
   Ans: Since $v \cdot dv/dt = \frac{1}{2}\frac{d|v|^2}{dt} = 1$, we have $|v|^2 = 2t + c$. We can determine that $c = 0$. Then $v(2) = 2$.

   At $(2,2)$, one vector parallel to the tangent line is $< 1, f'(x) >=< 1, 1/\sqrt{2x} > |_2 =< 1, 1/2 >$. The velocity is thus $2* < 1, 1/2 > *2/\sqrt{5} =< 4, 2 > /\sqrt{5}$