Keys to Quiz10

By Lei November 18, 2010

1. Find the center and the radius of the sphere $x^2 + y^2 + z^2 - 6y + 8z = 0$ $(2')$ and the midpoint between the point where the sphere meets the x-axis and the center $(1')$.
   Ans: Completing the square, we'll have the equation of the sphere as $x^2 + (y-3)^2 + (z+4)^2 = 25$. The center should be $(0, 3, -4)$ and the radius is $\sqrt{RHS} = \sqrt{25} = 5$ if it is a sphere and the Left Hand Side is the sum of several squares. Generally, the midpoint between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2})$. The point where the sphere meets the axis should satisfy both equations. The x-axis is $y = 0, z = 0$ and hence, the point is like $(a, 0, 0)$. It's on the sphere, so $a^2 + 0 + 0 - 0 + 0 = 0$ and thus $a = 0$. We have only one solution, which means the x-axis is tangent to the sphere. The midpoint is $(0, 1.5, -2)$.
   Qs: How about if we have two solutions or no solutions? Is it possible to have more than two solutions?

2. $\overrightarrow{AB} =< 1, 2, 4 >$, O is the origin and $A(0, 0, 1)$.
   For $\overrightarrow{AB}$, write it as the magnitude times the direction. $(1')$
   Ans: The magnitude is $\sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$ and the answer is $\sqrt{21} < \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}} >$ or equivalently $\sqrt{21}(\frac{1}{\sqrt{21}}i + \frac{2}{\sqrt{21}}j + \frac{4}{\sqrt{21}}k)$

   Find the angle between $\overrightarrow{OB}$ and $\overrightarrow{AB}$ $(2')$ and the projection of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$ $(2')$
   Ans: Since the components of the vector equals the coordinate of the termial point minus the initial point, we can figure out that $B(1, 2, 5)$. $\overrightarrow{OB} =< 1, 2, 5 >$. The angle should be $\cos^{-1}(\frac{<1,2,4>\cdot<1,2,5>}{\sqrt{21}\sqrt{30}}) = \cos^{-1}(\frac{25}{3\sqrt{70}})$. The projection should be $\frac{\overrightarrow{OB}\cdot\overrightarrow{OA}}{|OA|^2}\overrightarrow{OA} =< 0, 0, 5 >= 5k$ By obervation, you can also get this, because it's just the z-component of the $\overrightarrow{OB}$

   Write $\overrightarrow{AB} - \frac{1}{2}\overrightarrow{OA}$ as a linear combination of $\overrightarrow{OB}$ and $\overrightarrow{OA}$ $(2')$
   Ans:The first method is to use $\overrightarrow{AB} = \overrightarrow{QB} - \overrightarrow{QA}$ for any point Q. Here, we can have the result is $\overrightarrow{OB} - \frac{3}{2}\overrightarrow{OA}$ if we let $Q = O$. Of course you can also calculate the coordinates of the vector and then get the equations and then solve.

   (Bonus) Find a point P between O and B such that AP is perpendicular to OB $(2')$
   Ans: I just want to test the concept of projection here. You can figure out that $\overrightarrow{OP}$ should be the projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$. Then you can do. Answer is $< 1/6, 2/6, 5/6 >$