Keys to Quiz1

September 10, 2010

Some formulas:
$\int \frac{1}{\sqrt{a^2-u^2}}\mathrm{d}u = \sin^{-1}(\frac{u}{a}) + C$ $\int \frac{1}{a^2+u^2}\mathrm{d}u = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$
$\int \frac{1}{u\sqrt{u^2-a^2}}\mathrm{d}u = \frac{1}{a}\sec^{-1}|\frac{u}{a}| + C$

1. Complete **ONE** of the formulas below.(3 points)
   $\int \sec u \tan u \mathrm{d}u$ $\int \tan u \mathrm{d}u$
   Note:Please refer to the book. I'd like to remind you to distinguish the difference between the derivative of $\tan u$ and the its integral.

2. Calculations. Find the integrals below. (7 points)

   (a) $\int_{\pi/4}^{\pi/3} \sec^2 y e^{\tan y}\mathrm{d}y$ (4 pts)
   Ans:We know the result of a definite integral is a number. Let's find the number. By substitution $u = \tan y$, we can get the antiderivative is $e^{\tan y} + C$. So we have the answer is $e^{\tan y}|_{\pi/4}^{\pi/3} = e^{\sqrt{3}} - e$.
   Writing in another way is like this:
   Let $u = \tan y$, we have $du = \sec^2 y dy$. When $y = \pi/4$, u=1 and when $y = \pi/3$, $u = \sqrt{3}$.
   So the integral becomes like this:
   $\int_1^{\sqrt{3}} e^u \mathrm{d}u = e^{\sqrt{3}} - e$. (pay attention to the upper limit and lower limit.)

   (b) $\int \frac{x^4}{1+x^2}\mathrm{d}x$ (3 pts)
   Ans: Notice the degree of the numerator isn't smaller than that of the denominator, so we should reduce this improper fraction.
   By long division,$\frac{x^4}{x^2+1} = x^2 - 1 + \frac{1}{x^2+1}$, so we have
   $\int \frac{x^4}{1+x^2}\mathrm{d}x = \int (x^2 - 1 + \frac{1}{x^2+1})\mathrm{d}x = \frac{x^3}{3} - x + \arctan x + C$

Extra problem

Instructions: This is the extra problem, whose points can add to the total score. However,the total score won't be more than 10 points.
Find a way to calculate $I_k = \int \tan^{2k+1} x \mathrm{d}x$, where k is a nonnegative integer.
Hint: Find the relationship between I_k and I_{k-1} when $k \geq 1$, and you may want to use the relationship between $\sec x$ and $\tan x$ (5 pts)
Ans:
$I_k = \int \tan^{2k+1} x \mathrm{d}x = \int \tan^{2k-1} x * \tan^2 x \mathrm{d}x = \int \tan^{2k-1} x * (\sec^2 x - 1)\mathrm{d}x = \int \tan^{2k-1} x \sec^2 x \mathrm{d}x - \int \tan^{2k-1} x \mathrm{d}x = \frac{1}{2k} \tan^{2k} x - I_{k-1}$
Attention: Since I_k and I_{k-1} both are the sets of some functions, we can omit the constant term here.
Thus, if we want to know I_k, we should know I_{k-1}. So we need to calculate I_{n_0} for some n_0, and we can see if $n_0 = 0$, we can do, which is $I_0 = \int \tan x \mathrm{d}x = \ln|\sec x| + C$.