Math 222 Keys and Hints for HW9
By Lei November 4, 2010

I didn’t write out the detailed solutions, because it would be a lot of typing for this section if I wrote detailedly.

Section 9.2
1–6,9,10,11,15,16,25,26,27
1. Ans:We can see that \((xy)’ = xy’ + y\), so we don’t need to multiply integrating factor. \(xy = e^x + C\) and thus \(y = e^x/x + C/x\).
2. Ans:Standard form \(y’ + 2y = e^{-x}\), and the integrating factor \(e^{\int 2dx}\). Choose \(\mu(x) = e^{2x}\). We have \(e^{2x}y’ + 2e^{2x}y = e^x\). Hence \(e^{2x}y = e^x + C\), which means \(y = e^{-x} + Ce^{-2x}\).
3. Ans:Divide by x first and we have the integrating factor as \(x\). Then we have \(x^3y’ + 3x^2y = \sin x\). We have \(x^3y = -\cos x + C\), which is equivalent to \(y = -\cos x/x^3 + C/x^3\)
4. Ans:It’s already of the standard form of first order linear equation. Integrating factor \(\mu(x) = \sec x\). (sec \(x\))’ = \(\cos x\). We have sec \(xy = \sin x + C\), which implies \(y = \sin x \cos x + C \cos x\)
5. Ans:Divide first by x and then multiply factor \(\mu(x) = x^2\). \(x^2y’ + 2xy = x - 1\).
\(y = \frac{(x-1)^2}{x^2} + \frac{C}{x^2}\)
6. Ans:We don’t need to multiply integrating factor. \(((1 + x)y)’ = \sqrt{x}\) and \(y = \frac{2x^{3/2}}{3(x+1)} + \frac{C}{x+1}\)
9. Ans:Divide by x first to get the standard form. Then multiply 1/x. We have \(y’/x - y/x^2 = 2 \ln x/x\). Then \(y/x = (\ln x)^2 + C\). Hence \(y = x(\ln x)^2 + Cx\)
10. Ans:x^2y’ + 2xy = \cos x. \(y = (\sin x + C)/x^2\)
11. Ans:((t - 1)^4s)’ = t^2 - 1. \(s = \frac{t^5/3 - t}{(t-1)^4} + \frac{C}{(t-1)^4}\)
15. Ans:The general solution is \(y = e^{-2t}((\frac{3}{2}e^{2t} + C)\). \(C = -1/2\)
16. Ans:General solution is \(y = \frac{t^4}{x} + \frac{C}{x^2}\). Then we can decide \(C = -12/5\)
25. Ans:
   a. \(b = 2 \cdot 5 = 10 \text{ lb/min.}\)
   b. \(V(t) = 100 + 5t - 4t = 100 + t \text{ gal.}\)
   c. \(y(t) / V(t) = \frac{4y}{100+t} \text{ lb/min}\)
   d. \(y(0) = 50 \text{ lb. The equation is } c'(t) = 10 - \frac{4y}{100+t}. \text{ We then can solve it as }\)
   \(((t + 100)^4y') = 10(t + 100)^4\). Hence \(y = 2(t + 100) + \frac{C}{(t+100)^4}\) and \(C = -150 \cdot 100^4\)
   e. concentration = \(y(25)/V(t) = [2 \cdot 125 - \frac{150}{(1.25)^4}] / 125 \text{ lb/gal.}\)
27. Ans:You use the similar method to get the equation. Let \(y’ = 0\), and you’ll get the amount \(y\). Then solve \(t\). I’d like to omit the solution here.

Section 17.1
1–4,16,17,31,32,37,42,43,46,57,59,60,65
1. Ans: Auxiliary equation: \( r^2 - r - 12 = 0 \). \( r_1 = 4, r_2 = -3 \). We have \( y(x) = C_1e^{4x} + C_2e^{-3x} \)
2. Ans: \( 3r^2 - r = 0 \). \( y(x) = C_1e^{x/3} + C_2 \)
3. Ans: \( y(x) = C_1e^{x} + C_2e^{-4x} \)
4. Ans: \( y(x) = C_1e^{3x} + C_2e^{-3x} \)
16. Ans: \( x^2 + 16 = 0 \) \( r = 4i \) or \( -4i \). \( y(x) = C_1\cos(4x) + C_2\sin(4x) \)
17. Ans: \( r^2 + 2r + 4 = 0 \) \( r = -1 \pm \sqrt{3}i \) \( y(x) = e^{-x}(C_1\cos(\sqrt{3}x) + C_2\sin(\sqrt{3}x)) \)
31. Ans: General solution \( y = C_1e^{-x} + C_2e^{-5x} \). We have \( C_1 + C_2 = 0 \) and \( -C_1 - 5C_2 = 3 \). We have \( y = \frac{3}{4}e^{-x} - \frac{3}{4}e^{-5x} \)
32. Ans: \( y(x) = 2\cos(4x) - \frac{1}{2}\sin(4x) \)
37. Ans: General solution \( y(x) = C_1e^{2x} + C_2xe^{2x} \) and the unique solution for this is \( y(x) = e^{2x}(1 - 2x) \)
42. Ans: \( y(x) = C_1e^{-x/3} + C_2e^{x/2} \)
33. Ans: \( (2r + 1)^2 = 0 \) and \( y(x) = C_1e^{-x/2} + C_2xe^{-x/2} \)
45. Ans: \( y(x) = e^{-x}(C_1\cos x + C_2\sin x) \)
57. Ans: \( y(x) = e^{-x} + 2xe^{-x} \)
59. Ans: \( y(x) = \frac{11}{13}e^{2x} + \frac{15}{13}e^{-7x/3} \).
64. Ans: General solution \( y(x) = e^{-x/2}(C_1\cos(x) + C_2\sin(x)) \).
65. Ans: General solution \( y(x) = C_1\cos(2x) + C_2\sin(2x) \). We can see that no matter what \( C_1 \) and \( C_2 \) are \( y(0) = y(\pi) \). Hence the first problem has no solution. For the second, we can only decide \( C_1 = 0 \).