Math 222 Keys and Hints for HW9

By Lei   November 4, 2010

I didn't write out the detailed solutions, because it would be a lot of typing for this section if I wrote detailedly.

Section 9.2

1–6,9,10,11,15,16,25,26,27

1. Ans:We can see that $(xy)' = xy' + y$, so we don't need to multiply integtating factor. $xy = e^x + C$ and thus $y = e^x/x + C/x$.

2. Ans:Standard form $y' + 2y = e^{-x}$, and the integrating factor $e^{\int 2dx}$. Choose $\mu(x) = e^{2x}$. We have $e^{2x}y' + 2e^{2x}y = e^x$. Hence $e^{2x}y = e^x + C$, which means $y = e^{-x} + Ce^{-2x}$.

3. Ans:Divide by x first and we have the integrating factor as x^3. Then we have $x^3y' + 3x^2y = \sin x$. We have $x^3y = -\cos x + C$, which is equivalent to $y = -\cos x/x^3 + C/x^3$

4. Ans:It's already of the standard form of first order linear equation. Integrating factor $\mu(x) = \sec x$. $(\sec xy)' = \cos x$. We have $\sec xy = \sin x + C$, whih implies $y = \sin x \cos x + C \cos x$

5. Ans:Divide first by x and then multiply factor $\mu(x) = x^2$. $x^2y' + 2xy = x - 1$. $y = \frac{(x-1)^2}{2x^2} + \frac{C}{x^2}$

6. Ans:We don't need to multiply integrating factor.$((1 + x)y)' = \sqrt{x}$ and $y = \frac{2x^{3/2}}{3(x+1)} + \frac{C}{x+1}$

9. Ans:Divide by x first to get the standard form. Then multiply $1/x$. We have $y'/x - y/x^2 = 2\ln x/x$. Then $y/x = (\ln x)^2 + C$. Hence $y = x(\ln x)^2 + Cx$

10. Ans:$x^2y' + 2xy = \cos x$. $y = (\sin x + C)/x^2$

11. Ans:$((t - 1)^4s)' = t^2 - 1$. $s = \frac{t^3/3-t}{(t-1)^4} + \frac{C}{(t-1)^4}$

15. Ans:The general solution is $y = e^{-2t}(\frac{3}{2}e^{2t} + C)$. $C = -1/2$

16. Ans:General solution is $y = \frac{t^3}{5} + \frac{C}{t^2}$. Then we can decide $C = -12/5$

25. Ans:

a. $b = 2 * 5 = 10$ lb/min.

b. $V(t) = 100 + 5t - 4t = 100 + t$ gal.

c. $y(t) * 4/V(t) = \frac{4y}{100+t}$ lb/min

d. $y(0) = 50$ lb.The equation is $c'(t) = 10 - \frac{4y}{100+t}$. We then can solve it as $((t + 100)^4y)' = 10(t + 100)^4$. Hence $y = 2(t + 100) + \frac{C}{(t+100)^4}$ and $C = -150 * 100^4$

e. $concentration = y(25)/V(t) = [2 * 125 - \frac{150}{(1.25)^4}]/125$ lb/gal.

26. Similar to 25. Omitted.

27. Ans:You use the similar method to get the equation. Let $y' = 0$, and you'll get the amount y. Then solve t. I'd like to omit the solution here.

Section 17.1

1–4,16,17,31,32,37,42,43,46,57,59,60,65

1. Ans:Auxiliary equation: $r^2 - r - 12 = 0$. $r_1 = 4, r_2 = -3$. We have $y(x) = C_1 e^{4x} + C_2 e^{-3x}$
2. Ans:$3r^2 - r = 0$. $y(x) = C_1 e^{x/3} + C_2$
3. Ans:$y(x) = C_1 e^x + C_2 e^{-4x}$
4. Ans:$y(x) = C_1 e^{3x} + C_2 e^{-3x}$
16. Ans:$r^2 + 16 = 0$ $r = 4i$ or $-4i$. $y(x) = C_1 \cos(4x) + C_2 \sin(4x)$
17. Ans:$r^2 + 2r + 4 = 0$ $r = -1 \pm \sqrt{3}i$ $y(x) = e^{-x}(C_1 \cos(\sqrt{3}x) + C_2 \sin(\sqrt{3}x))$
31. Ans:General sol $y = C_1 e^{-x} + C_2 e^{-5x}$. We have $C_1 + C_2 = 0$ and $-C_1 - 5C_2 = 3$. We have $y = \frac{3}{4}e^{-x} - \frac{3}{4}e^{-5x}$
32. Ans:$y(x) = 2\cos(4x) - \frac{1}{2}\sin(4x)$
37. Ans:General sol $y(x) = C_1 e^{2x} + C_2 x e^{2x}$ and the unique sol for this is $y(x) = e^{2x}(1 - 2x)$
42. Ans:$y(x) = C_1 e^{-x/3} + C_2 e^{x/2}$
43. Ans:$(2r + 1)^2 = 0$ and $y(x) = C_1 e^{-x/2} + C_2 x e^{-x/2}$
46. Ans:$y(x) = e^{-x}(C_1 \cos x + C_2 \sin x)$
57. Ans:$y(x) = e^{-x} + 2xe^{-x}$
59. Ans:$y(x) = \frac{11}{13}e^{2x} + \frac{15}{13}e^{-7x/3}$.
60. Ans:General solution $y(x) = e^{-x/2}(C_1 \cos(x) + C_2 \sin(x))$.
$y(x) = e^{-x/2}(-e^{\pi/2}\cos(x) - \frac{1}{2}e^{\pi/2}\sin(x))$
65. Ans:General solution $y(x) = C_1 \cos(2x) + C_2 \sin(2x)$. We can see that no matter what C_1 and C_2 are $y(0) = y(\pi)$. Hence the first problem has no solution. For the second, we can only decide $C_1 = 0$.